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EXCHANGE RATE DYNAMICS WITH SLUGGISH PRICES UNDER ALTERNATIVE PRICE-ADJUSTMENT RULES*

BY MAURICE OBSTFELD AND KENNETH ROGOFF[1]

1. INTRODUCTION

Recent research on the asset-market approach to exchange rates has incorporated short-run Keynesian price rigidities into models assuming rational expectations. These sticky-price models generally exhibit classical properties in the long run, but allow for temporary goods-market disequilibrium in response to real and monetary shocks that are less than perfectly anticipated. A critical element in these models is the mechanism determining how domestic goods prices adjust over time in response both to current disequilibrium and to expectations of future events.

In his seminal paper on exchange-rate determination with sticky domestic prices, Dornbusch [1976] assumed that the nominal price of domestic output is a predetermined variable that moves only in response to current goods-market disequilibrium. Mussa [1977, 1982] has criticized the simple Dornbusch adjustment rule as being inadequate in situations where future disturbances are anticipated or where the long-run equilibrium of the economy moves over time. Frankel [1979], Liviatan [1980], and Buiter and Miller [1981, 1982] introduce trend inflation into the Dornbusch model by linking price adjustment to the underlying (constant) money growth rate in addition to direct goods-market pressure. An alternative price-adjustment scheme allowing for very general moving long-run equilibria is derived by Mussa [1981].

The paper compares the price-adjustment rule of Mussa [1981] to a rule advanced by Barro and Grossman [1976] in a closed-economy context. The interest of the Barro-Grossman rule is twofold. First, like Mussa's rule, that of Barro and Grossman is appropriate in models with anticipated future disturbances or nonstationary long-run equilibria. Second, the Barro-Grossman rule contains the rules of Dornbusch, Frankel, Liviatan, and Buiter and Miller as special cases, and thus, has an intuitive interpretation.

The paper's central result is that the Mussa and Barro-Grossman rules, though apparently quite dissimilar, yield structurally equivalent exchange rate models.[2]

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[1] This paper represents the views of its' authors and not those of the Board of Governors of the Federal Reserve System. Obstfeld acknowledges with thanks the financial support of the National Science Foundation.

[2] The models explored below assume that agents have perfect foresight, but the structural equivalence result would carry over to an explicitly stochastic environment such as the one assumed by Mussa [1982]. In a stochastic setting, structural equivalence is the same as econo-

(*Continued on next page*)

Thus, despite the key role of disequilibrium price dynamics, the choice between the two adjustment mechanisms is not necessarily a critical one.

2. THE LIMITING FLEXIBLE-PRICE MODEL

The dynamics of a sticky-price exchange rate model with rational expectations can be decomposed into two components. The first component is caused by the system's adjustment to current disequilibrium. The second component is caused by movement of the equilibrium that would obtain if all prices were fully flexible.[3] Perfectly predictable trend movements in the money supply or in the equilibrium terms of trade, for example, cause no disequilibrium in a well-specified model, but do induce movements of the system. In this section, we focus on the second source of dynamics by solving a standard exchange rate model under the temporary assumption that domestic prices are fully flexible, or, alternatively, that all movements in the exogenous variables affecting the economy are perfectly anticipated. The equilibrium path of this flexible-price model provides a limiting benchmark for the sticky-price models analyzed later. This benchmark is a natural generalization of the fixed "long-run" equilibrium appearing in sticky-price models whose exogenous variables are static except for one-time unexpected jumps.

The exchange-rate model used here is of the extended small-country variety, and is based on work of Dornbusch [1976] and Mussa [1977, 1982].[4] It is described by the following equations:

(1) $m^d - \alpha p - (1-\alpha)e = -\lambda r + \psi y$ (money demand)

(2) $r = r^* + \dot{e}$ (uncovered interest parity)

(3) $y^d = \phi(e-p+u) - \sigma[r-\alpha\dot{p}-(1-\alpha)\dot{e}] + \gamma y$ (aggregate demand)

(4) $m^d = m$ (money-market equilibrium)

(5) $y^d = y$ (goods-market equilibrium).

Here, p is the logarithm of the money price of domestically-produced goods; e is the logarithm of the exchange rate (defined as the domestic-currency price of foreign currency); and r is the domestic nominal interest rate. Dots over variables indicate rates of change.[5] The remaining variables are exogenous: m is the

(*Continued*)
metric observational equivalence. The "perfect foresight" assumption allows for the initial arrival of previously unanticipated information concerning the future paths of relevant exogenous variables. After the initial moment, however, agents' expectations are fulfilled.

[3] Even if prices are flexible and exogenous variables are stationary, dynamics may arise from the adjustments of nonmonetary asset stocks to long-run desired levels, as in Kouri [1976]. The class of models studied in this paper abstracts from the dynamics of asset accumulation. For an analysis of asset accumulation in a sticky-price model, see Henderson [1980].

[4] See the Dornbusch and Mussa papers for more detailed expositions.

[5] Unless otherwise stated, these rates of change are right-hand derivatives.

logarithm of the nominal money supply; α is the share of home goods in the domestic consumer price index; y is the logarithm of the flow of perishable home output; r^* is the nominal interest rate on foreign-currency bonds; and u is a shock to foreign demand for domestic output. α is assumed to be fixed, and y and r^* are assumed to be fixed and equal to zero. The exogenous foreign-currency price of imports is likewise assumed to be constant, with its natural logarithm normalized to zero.

The model assumes rational expectations; this amounts to perfect foresight in the absence of unanticipated shocks. Thus, there is no distinction between actual and anticipated rates of change of e and p. Equation (1) is the money-demand schedule, which relates the demand for price-index deflated nominal balances to the home interest rate and income. Equation (2) reflects the assumption that home-currency and foreign-currency bonds are perfect substitutes. Equation (3) is the aggregate-demand schedule, which posits that demand for domestic output depends on the terms of trade, tastes (as represented by u), the *real* rate of interest, and income. Finally, equations (4) and (5) require continuous clearing of both the money and goods markets. Condition (5), which entails full flexibility of domestic prices, will be relaxed in subsequent sections.

The flex-price equilibrium values of the exchange rate and domestic output price are denoted by $\tilde{e}$ and $\tilde{p}$. Following Sargent and Wallace [1973], we close the model with the assumption that the economy must always lie on its unique conditionally stable saddlepath.[6] As the Appendix shows, the rational-expectations solution paths for $\tilde{e}$ and $\tilde{p}$ are then given by

$$(6) \qquad \tilde{e}_t = \frac{1}{\lambda}\int_t^{\infty} \exp\left[(t-s)/\lambda\right] m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_t^{\infty} \{\exp\left[(t-s)/\lambda\right] - \exp\left[\omega(t-s)\right]\} u_s ds$$

$$(7) \qquad \tilde{p}_t = \frac{1}{\lambda}\int_t^{\infty} \exp\left[(t-s)/\lambda\right] m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_t^{\infty} \exp\left[(t-s)/\lambda\right] u_s ds + \frac{\omega[(1-\alpha)-\lambda\omega]}{1-\lambda\omega}\int_t^{\infty} \exp\left[\omega(t-s)\right] u_s ds,$$

where $\omega \equiv \phi/\sigma\alpha$.[7] Under rational expectations, equilibrium nominal prices depend on the future expected paths of money and the autonomous component

[6] In models with a well-defined stationary long-run equilibrium, the saddlepath is the unique path converging to long-run equilibrium and thus is the unique path implying nonexplosive behavior for the economy. Saddlepoint stability results from the self-fulfilling nature of rational expectations. Whether there is a stationary long-run equilibrium or a moving long-run equilibrium, the saddlepath can be defined as the unique path of the economy along which prices do not depend in part on pure speculative "bubbles" unrelated to actual market conditions. The appendix discusses the mathematical implementation of this definition in the present context.

[7] These solutions are derived by the method of Laplace transforms. The Laplace transform technique is a convenient one for rational-expectations models, although other solution techniques are available.

of aggregate demand. Note that monetary factors affect $\tilde{e}_t$ and $\tilde{p}_t$ by equal amounts, whereas demand shifts affect these variables differentially in general. Thus, while real shocks must be accommodated by shifts in the flex-price real exchange rate $\tilde{q}_t \equiv \tilde{e}_t - \tilde{p}_t$, $\tilde{q}_t$ depends exclusively on factors that shift aggregate demand, y^d, and not on money. In the flex-price model set out here, there is a complete dichotomy between real and monetary phenomena. This classical dichotomy would disappear, for example, if real balances $m_t - \alpha p_t - (1-\alpha)e_t$ were an additional argument in the aggregate demand function. To simplify the analysis, we abstract from real balance effects.

3. STICKY DOMESTIC PRICES AND ALTERNATIVE PRICE-ADJUSTMENT RULES

When the price of domestic output is sticky, the goods-market clearing condition (5) need not hold. Following Dornbusch [1976], we assume that the domestic output price is a predetermined or nonjumping variable that adjusts gradually to eliminate goods-market disequilibrium. Dornbusch, who assumes that the exogenous variables are constant except for one-time unanticipated jumps, postulates the following price-adjustment scheme:

$$\dot{p} = \pi(y^d - y). \tag{8}$$

While specification (8) is entirely appropriate given the environment Dornbusch assumes, it may become inappropriate once anticipated shocks or trend movements in exogenous variables are introduced.[8]

Gray and Turnovsky [1979] and Wilson [1979] use the Dornbusch model with adjustment rule (8) to analyze a one-time anticipated increase in the money supply. The solution these authors derive is qualitatively sensible when the disturbance is expected to occur in the near future, but problems arise when (8) is used to study money shocks that are anticipated long before they occur. To see this, consider the economy's behavior as the date of the future money increase recedes infinitely far into the future. As the adjustment period preceding the intended policy act grows longer, the disequilibrium caused by the announcement of that act should disappear. In the limit of a perfectly anticipated money-stock increase — one that is anticipated "infinitely far" in advance — the domestic output price and the exchange rate should rise gradually and in proportion toward the long-run equilibrium levels associated with the post-disturbance stock of money.[9] How-

[8] Mussa [1982] has emphasized these problems.

[9] Prices naturally exhibit this behavior in the flex-price model of the previous section. When the money shock is expected to occur in finite time and prices are flexible, the domestic output price and exchange rate, after an initial equiproportionate jump when the announcement is made, rise smoothly and in proportion so that the economy is at its long-run equilibrium when the money supply increases. This type of adjustment to imperfectly anticipated shocks is impossible in the sticky-price setting, but it is still true that, after the initial announcement, price evolves smoothly. In particular, the exchange rate cannot jump when the expected money increase takes place, for an anticipated discrete jump would entail an unexploited opportunity to earn an infinite instantaneous rate of return on foreign bonds.

ever, this equilibrium scenario is impossible with adjustment-rule (8), because the price level cannot rise in the absence of excess demand. Even perfectly anticipated money shocks must cause goods-market disequilibrium.[10]

Similar difficulties surround the Frankel [1979], Liviatan [1980], and Buiter-Miller [1981, 1982] extensions of Dornbusch's price-adjustment rule to environments of secular inflation.[11] These authors modify (8) by adding to the excess demand term the current rate of nominal money growth. The resulting adjustment rule is

$$(9) \qquad \dot{p}_{\circ} = \pi(y^d - y) + \dot{m}.$$

Specification (9) is consistent with an assumption that all changes in the monetary growth rate are unanticipated. But the rule implies unreasonable asymptotic behavior when anticipated changes are analyzed. Consider, for example, a perfectly anticipated increase in the monetary growth rate. With higher trend money growth, long-run $m-p$ is lower. Thus, a perfectly anticipated money-growth increase should cause $m-p$ to decline over time but should not occasion disequilibrium. Because p is predetermined, however, rule (9) implies that $m-p$ can fall over time only if the announcement of the future increase in $\dot{m}$ creates an excess demand for output that does not vanish as the date of the increase becomes infinitely distant.

While the price-adjustment rules described so far are unsatisfactory in many situations, a generalization which contains these rules as special cases is adequate for the analysis of any type of disturbance. Assume now that $\dot{p}$ is a function not only of current disequilibrium, but also of the rate at which the price of domestic output would increase if that price were fully flexible. The resulting price adjustment scheme is

$$(10) \qquad \dot{p} = \pi(y^d - y) + \dot{\tilde{p}},$$

where $\tilde{p}$ is the flex-price equilibrium output price discussed in the previous section. This type of pricing rule is suggested by Barro and Grossman [1976] in a closed-economy setting, although they do not assume rational expectations regarding $\dot{\tilde{p}}$.[12] It is easy to see that the Barro-Grossman rule reduces to Dornbusch's rule (8) when the exogenous variables are fixed except for unanticipated jumps,

[10] To be precise, the problems with the Gray-Turnovsky and Wilson analyses are not caused exclusively by the price-adjustment scheme these authors adopt. An additional source of nonneutrality is their assumption that aggregate demand is a function of the nominal, rather than the real, domestic interest rate. It is easy to see that in a well-specified model, the nominal interest rate would rise over time during the adjustment to a perfectly anticipated money increase while the real interest rate would remain constant.

[11] Like Dornbusch [1976], Section 5, Buiter and Miller analyze a model in which output is demand-determined and therefore endogenous. In that model, the disequilibrium term entering their price adjustment rule depends on the difference between actual output and full-employment potential output. Our analysis applies with only minor modifications to variable-output models; see footnote 15 below.

[12] See Barro and Grossman [1976, p. 178], equation (4.26).

for in that case, $\dot{\tilde{p}}=0$. Similarly, the rule reduces to (9) when the only expected change in the exogenous variables is the growth of money at a constant rate. However, the rule does *not* reduce to Dornbusch's rule (8) in the case of the one-time expected money increase analyzed by Gray and Turnovsky [1979] and by Wilson [1979]. If it is announced at time $t=0$ that the money stock will increase by an amount Δm at time $t=T$, then $\dot{\tilde{p}}_t=(1/\lambda)\Delta m \exp[(t-T)/\lambda]>0$ for $0\leq t<T$.

That the Barro-Grossman rule is immune to the criticisms levelled at rules (8) and (9) is clear. Perfectly anticipated shocks — whether real or nominal — do not cause disequilibrium when price adjustment is given by (10). Since the equilibrium flexible-price $\tilde{p}$ does not jump discontinuously at *any* point in response to a change announced "infinitely far" in advance, p will fully trace its movements under (10).

Mussa [1977, 1982] suggests an alternative rule, and demonstrates that it, too, renders disequilibrium price adjustment unnecessary when shocks are perfectly anticipated. The Mussa rule is given by

$$\dot{p} = \theta(y^d - y) + \dot{\bar{p}} \tag{11}$$

where $\bar{p}$ is defined as the domestic output price that would clear the goods market given the actual (possibly disequilibrium) values of the endogenous variables e, r, $\dot{e}$, and $\dot{p}$. More formally, $\bar{p}_t$ is defined by the condition

$$y = 0 = \phi(e_t - \bar{p}_t + u_t) - \sigma[r_t - \alpha\dot{p}_t - (1-\alpha)\dot{e}_t]. \tag{12}$$

The difference between $\bar{p}$ and $\tilde{p}$ deserves emphasis. $\tilde{p}$ is the output price that would prevail in a hypothetical Walrasian general equilibrium with fully flexible prices. $\bar{p}$ is the output price that would clear the goods market given current levels of the sticky-price system's endogenous variables.

Alternative microeconomic rationales for the Mussa price-adjustment rule are presented in McCallum [1980], Mussa [1981], and Flood [1982b]. Mussa's derivation, for example, assumes monopolistic firms for whom price changes are costly. Flood's inventory-adjustment story assumes that firms set their prices a period in advance of market transactions.[13]

The Barro-Grossman rule (10) and the Mussa rule (11) appear quite unrelated, and it is natural to ask how these different price-adjustment schemes affect the dynamic behavior of the economy. We address this question in the next section by comparing explicit solutions of models incorporating the two rules.

4. THE STRUCTURAL EQUIVALENCE OF THE BARRO-GROSSMAN AND MUSSA PRICE-ADJUSTMENT RULES

The following result clarifies the relationship between the Barro-Grossman and Mussa pricing rules:

[13] Flood's justification of the Mussa rule is based on work of Green and Laffont [1981]. Rotemberg [1981] points out some limitations of Mussa's [1981] rationale.

THEOREM. *Provided that the condition*

$$1 - \pi\alpha\sigma > 0 \tag{13}$$

is satisfied, the Barro-Grossman price adjustment scheme (10) *and the Mussa price adjustment scheme* (11) *yield structurally equivalent exchange rate models.*

PROOF. The Appendix demonstrates [see equations (A18) and (A19)] that when (13) holds, the rational-expectations equilibrium of the model described by equations (1) through (4) and the Barro-Grossman rule (10) is given by

$$e_t^{BG} = \frac{-(p_0 - \tilde{p}_0)\alpha}{[(1-\alpha) - \lambda\eta_2]} \exp(\eta_2 t) + \tilde{e}_t \tag{14a}$$

$$p_t^{BG} = (p_0 - \tilde{p}_0) \exp(\eta_2 t) + \tilde{p}_t, \tag{14b}$$

where

$$\eta_2 = \frac{-[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]}{2\lambda(1-\pi\alpha\sigma)} - \left\{\frac{[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]^2}{4\lambda^2(1-\pi\alpha\sigma)^2} + \frac{\pi\phi}{(1-\pi\alpha\sigma)}\right\}^{1/2} < 0, \tag{15}$$

and $\tilde{e}_t$ and $\tilde{p}_t$ again denote the equilibrium solutions for the flex-price model of section 2. When the Mussa rule (11) is substituted for (10), the rational-expectations equilibrium is given by

$$e_t^{M} = \frac{-(p_0 - \tilde{p}_0)\alpha}{[(1-\alpha) + \lambda\theta\phi]} \exp(-\theta\phi t) + \tilde{e}_t \tag{16a}$$

$$p_t^{M} = (p_0 - \tilde{p}_0) \exp(-\theta\phi t) + \tilde{p}_t \tag{16b}$$

[see the appendix, equations (A35) and (A36)]. The two sets of solutions can be made numerically equal by choosing θ (the speed-of-adjustment parameter in the Mussa model) so that

$$\theta = \frac{[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]}{2\phi\lambda(1-\pi\alpha\sigma)} + \left\{\frac{[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]^2}{4(\phi\lambda)^2(1-\pi\alpha\sigma)^2} + \frac{\pi}{\phi\lambda(1-\pi\alpha\sigma)}\right\}^{1/2}, \tag{17}$$

for when θ is so chosen, $-\theta\phi = \eta_2$ [cf. (15)]. The two models are therefore structurally equivalent.

It is important to note that the theorem holds only along the saddlepath. The models are not structurally equivalent elsewhere.

When condition (13) is violated (as it necessarily is for π sufficiently large), the model with the Barro-Grossman adjustment rule has two characteristic roots with positive real part. Saddlepath stability requires that a negative root be associated with each predetermined variable in the model; and because p is predetermined, a rational-expectations equilibrium will not exist in general when (13) fails and the model is unstable.[14] In contrast, the Mussa rule necessarily yields saddlepath stability for any nonnegative values of the system's parameters; and as the speed-of-adjustment parameter $\theta \to +\infty$, the Mussa model converges smoothly to the flex-price model. It seems worrisome, at first glance, that under the Barro-Grossman rule, the economy becomes unstable and does not converge to the flex-price model as $\pi \to +\infty$. We will argue shortly that when the price adjustment implied by the Barro-Grossman mechanism is interpreted properly, (13) always holds and this apparent convergence problem disappears.

The following result makes apparent the essential reason for the two models' structural equivalence.

COROLLARY. *Along the saddlepath of the Mussa model, the output price p obeys a differential equation having the same form as the Barro-Grossman equation* (10). *More precisely, along the saddlepath of the Mussa model,*

$$\dot{p}^M = \delta(y^d - y) + \dot{\tilde{p}} \tag{18}$$

where

$$\delta = \frac{\theta[(1-\alpha)+\lambda\theta\phi]}{(1+\theta\alpha\sigma)(1+\lambda\theta\phi)}. \tag{19}$$

PROOF. Letting q_t once again denote the real exchange rate $e_t - p_t$, we may use (3) to write excess demand in the Mussa model as

$$y_t^d - y = \phi(q_t^M - \tilde{q}_t) - \alpha\sigma(\dot{q}_t^M - \dot{\tilde{q}}_t). \tag{20}$$

Equation (A28) of the Appendix implies that on the saddlepath of the Mussa model,

$$\dot{q}_t^M - \dot{\tilde{q}}_t = -\theta\phi(q_t^M - \tilde{q}_t). \tag{21}$$

Equations (20) and (21) together imply that

$$y_t^d - y = \phi(1+\theta\alpha\sigma)(q_t^M - \tilde{q}_t) = \phi(1+\theta\alpha\sigma)[(e_t^M - \tilde{e}_t) - (p_t^M - \tilde{p}_t)]. \tag{22}$$

Using equations (16a) and (16b), we have

$$e_t^M - \tilde{e}_t = \frac{-\alpha(p_t^M - \tilde{p}_t)}{[(1-\alpha)+\lambda\theta\phi]}. \tag{23}$$

Combining (22) and (23) yields

$$p_t^M - \tilde{p}_t = \frac{-[(1-\alpha)+\lambda\theta\phi]}{\phi(1+\theta\alpha\sigma)(1+\lambda\theta\phi)}(y_t^d - y). \tag{24}$$

Because, by (16b), $\dot{p}_t^M = -\theta\phi(p_t^M - \tilde{p}_t) + \dot{\tilde{p}}_t$ along the Mussa model's saddlepath, (24) implies that

$$\dot{p}_t^M = \frac{\theta[(1-\alpha)+\lambda\theta\phi]}{(1+\theta\alpha\sigma)(1+\lambda\theta\phi)}(y_t^d - y) + \dot{\tilde{p}}_t.$$

[14] The instability is due to the fact that when (13) does not hold, a positive shock to aggregate demand (an increase in u) leads, *ceteris paribus*, to falling prices.

The foregoing corollary shows why the Mussa model is necessarily saddlepath stable. Although prices in that model do adjust according to a Barro-Grossman-type rule, the model can never be unstable because, by (19), the stability criterion

$$1-\delta\alpha\sigma>0 \tag{25}$$

is always satisfied [cf. equation (13)]. By assuming the Mussa rule, we effectively limit the parameter δ to values between 0 ($\theta=0$) and $1/\alpha\sigma$ ($\theta=+\infty$). Even though the Mussa rule places an upper bound on the excess-demand coefficient δ appearing in (18), there is no upper bound on the speed at which goods-market disequilibrium is eliminated. As was pointed out above, the Mussa model converges to the equilibrium flex-price model as $\theta\to+\infty$ and $\delta\to 1/\alpha\sigma$.

A consequence of these findings is that the speed of goods-market adjustment under the Barro-Grossman rule (10) becomes infinite as $\pi\to 1/\alpha\sigma$. Accordingly, that rule yields a saddlepoint-stable model for *any* speed of goods-market adjustment. Contrary to appearances, the stability condition (13) does not limit one to adjustment speeds which are not too great.[15]

5. CONCLUSION

This paper has studied the consequences of adopting alternative sticky-price adjustment rules in exchange rate models characterized by moving long-run equilibria. A price-adjustment scheme suggested by Barro and Grossman [1976] in a different context was shown to be a natural generalization of less versatile price-adjustment schemes advanced by Dornbusch [1976], by Frankel [1979], by Liviatan [1980], and by Buiter and Miller [1981, 1982]. It was also demonstrated that use of the Barro-Grossman rule results in an exchange rate model that is structurally equivalent to one based on the apparently quite different price-adjustment rule proposed by Mussa [1977, 1982]. The choice between the Barro-Grossman and Mussa rules is therefore not critical for many theoretical and empirical applications.[16] Unlike the simpler price-adjustment rules used in earlier studies, either yields sensible results for *any* expected path of the exogenous variables driving the system.

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[15] The isomorphism results presented here are readily extended to variable-output versions of the exchange rate model, like that of Buiter and Miller [1981, 1982] and those compared by Flood [1982a]. The stability condition when the Barro-Grossman mechanism is used is slightly more complicated in the variable-output case if money demand depends on actual output. The stability criterion, in that case, is $\lambda(1-\gamma)+\alpha\sigma\psi-\lambda\pi\alpha\sigma>0$. Otherwise, the isomorphism between variable-output models incorporating the Barro-Grossman rule and those incorporating the Mussa rule can be proved as in the text.

[16] As Lucas [1976] argues, changes of policy regime could alter the parameters appearing in the two rules. In some cases it may be easier to model this possibility using the Mussa rule, which at present has a somewhat better-developed microeconomic rationale.

APPENDIX

This appendix calculates explicit solution paths for the flexible-price exchange rate model and the two sticky-price models discussed in the text. These rational-expectations models are solved by the method of Laplace transforms, which is equivalent to the operator solution procedure described by Sargent [1979].[1] The Laplace transform of a function f_t is defined by

$$L(f_t) = \int_0^\infty \exp(-lt) f_t dt, \tag{A1}$$

and is a function of l.[2] The key theorem invoked below is that a continuous function is uniquely determined on $(0, \infty)$ by its Laplace transform (see Sokolnikoff and Redheffer [1966]). It is easy to verify that: (i) $L(\cdot)$ is linear; (ii) $L(\dot{f}_t) = lL(f_t) - f_0$; (iii) $L(f_t)L(g_t) = L\left(\int_0^t f_s g_{t-s} ds\right)$. These three properties will be used repeatedly in what follows.

1. *The Flex-Price Model*

The flex-price model may be written in the form

$$\dot{e}_t = [(1-\alpha)/\lambda]e_t + (\alpha/\lambda)p_t - m_t/\lambda \tag{A2}$$

$$\dot{p}_t = -\omega e_t + \omega p_t + \dot{e}_t - \omega u_t \tag{A3}$$

where $\omega \equiv \phi/\sigma\alpha$. The Laplace transform of the system given by (A2) and (A3) may be written in matrix notation as

$$\begin{bmatrix} l - \dfrac{(1-\alpha)}{\lambda} & -\dfrac{\alpha}{\lambda} \\ \omega - \dfrac{(1-\alpha)}{\lambda} & l - \dfrac{\alpha}{\lambda} - \omega \end{bmatrix} \begin{bmatrix} L(e_t) \\ L(p_t) \end{bmatrix} = \begin{bmatrix} e_0 - \dfrac{L(m_t)}{\lambda} \\ p_0 - \dfrac{L(m_t)}{\lambda} - \omega L(u_t) \end{bmatrix}.$$

By solving these simultaneous equations we obtain

$$L(e_t) = \frac{[\lambda(l-\omega)-\alpha]e_0 + \alpha p_0 + (\omega - l)L(m_t) - \alpha\omega L(u_t)}{\lambda[l-(1/\lambda)][l-\omega]}, \tag{A4}$$

$$L(p_t) = \frac{[(1-\alpha)-\lambda\omega]e_0 + [\lambda l - (1-\alpha)]p_0 + (\omega - l)L(m_t) + \omega[(1-\alpha)-\lambda l]L(u_t)}{\lambda[l-(1/\lambda)][l-\omega]}. \tag{A5}$$

A partial-fraction expansion of (A4) leads to the representation

[1] The advantages of the Laplace transform method are that it is completely algorithmic and that it produces solutions which are expressed in terms of the state variables' initial positions.

[2] The transform is defined only for l such that the integral in (A1) converges.

$$\text{(A6)}\quad L(e_t) = \frac{\alpha p_0 - (\alpha + \lambda\omega)e_0}{\lambda[(1/\lambda) - \omega]}\left[\frac{1}{l-(1/\lambda)} - \frac{1}{l-\omega}\right]$$
$$+ \frac{\lambda e_0}{\lambda[(1/\lambda) - \omega]}\left[\frac{(1/\lambda)}{l-(1/\lambda)} - \frac{\omega}{l-\omega}\right]$$
$$+ \frac{\omega}{\lambda[(1/\lambda) - \omega]} L\left(\int_0^t \{\exp[(t-s)/\lambda] - \exp[\omega(t-s)]\} m_s ds\right)$$
$$- \frac{1}{\lambda[(1/\lambda) - \omega]} L\left(\int_0^t \{(1/\lambda)\exp[(t-s)/\lambda] - \omega \exp[\omega(t-s)]\} m_s ds\right)$$
$$- \frac{\alpha\omega}{\lambda[(1/\lambda) - \omega]} L\left(\int_0^t \{\exp[(t-s)/\lambda] - \exp[\omega(t-s)]\} u_s ds\right).$$

In deriving (A6), we have used the convolution property (iii) and the facts that $L[\exp(t/\lambda)] = [l-(1/\lambda)]^{-1}$, $L[\exp(\omega t)] = (l-\omega)^{-1}$.

The Laplace transform theorem allows us to infer from (A6) that the exchange rate path has the form

$$\text{(A7)}\quad e_t = \frac{(1-\lambda\omega-\alpha)e_0 + \alpha p_0}{1-\lambda\omega}\exp(t/\lambda) + \frac{\alpha(e_0 - p_0)}{1-\lambda\omega}\exp(\omega t)$$
$$- \frac{1}{\lambda}\int_0^t \exp[(t-s)/\lambda] m_s ds - \frac{\alpha\omega}{1-\lambda\omega}\int_0^t \{\exp[(t-s)/\lambda]$$
$$- \exp[\omega(t-s)]\} u_s ds.$$

It is convenient to rewrite (A7) in the equivalent form

$$\text{(A8)}\quad e_t =$$
$$\frac{\left\{(1-\lambda\omega-\alpha)e_0 + \alpha p_0 - [(1/\lambda)-\omega]\int_0^\infty \exp(-s/\lambda) m_s ds - \alpha\omega\int_0^\infty \exp(-s/\lambda) u_s ds\right\}\cdot}{1-\lambda\omega}$$
$$\exp(t/\lambda) + \frac{[\alpha(e_0 - p_0) + \alpha\omega\int_0^\infty \exp(-\omega s) u_s ds]}{1-\lambda\omega}\exp(\omega t)$$
$$+ \frac{1}{\lambda}\int_t^\infty \exp[(t-s)/\lambda] m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_t^\infty \{\exp[(t-s)/\lambda] - \exp[\omega(t-s)]\} u_s ds.$$

Equation (A8) expresses the path of the exchange rate as a function of its own initial value (e_0) and the initial value of the price of domestic output (p_0). These two initial conditions are uniquely determined by the saddlepath assumption, which requires that the coefficients of the explosive "bubble" terms, $\exp(t/\lambda)$ and $\exp(\omega t)$, be zero (see Sargent and Wallace [1973]). The requirement that the economy be on its saddlepath implies [by (A8)] that

$$\text{(A9)}\quad \tilde{e}_0 = \frac{1}{\lambda}\int_0^\infty \exp(-s/\lambda) m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_0^\infty \{\exp(-s/\lambda) - \exp(-\omega s)\} u_s ds,$$

$$\text{(A10)} \quad \tilde{p}_0 = \frac{1}{\lambda}\int_0^\infty \exp(-s/\lambda)m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_0^\infty \exp(-s/\lambda)u_s ds + \frac{\omega[(1-\alpha)-\lambda\omega]}{1-\lambda\omega}\int_0^\infty \exp(-\omega s)u_s ds,$$

where a tilde denotes a saddlepath equilibrium value for the flex-price model. Combining (A8), (A9), and (A10), we obtain the rational-expectations equilibrium value of the exchange rate

$$\text{(A11)} \quad \tilde{e}_t = \frac{1}{\lambda}\int_t^\infty \exp[(t-s)/\lambda]m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_t^\infty \{\exp[(t-s)/\lambda] - \exp[\omega(t-s)]\}u_s ds.$$

A derivation similar to the foregoing shows that the flex-price equilibrium domestic output price can be written as

$$\text{(A12)} \quad \tilde{p}_t = \frac{1}{\lambda}\int_t^\infty \exp[(t-s)/\lambda]m_s ds + \frac{\alpha\omega}{1-\lambda\omega}\int_t^\infty \exp[(t-s)/\lambda]u_s ds + \frac{\omega[(1-\alpha)-\lambda\omega]}{1-\lambda\omega}\int_t^\infty \exp[\omega(t-s)]u_s ds.$$

According to (A11) and (A12), the flex-price exchange rate and domestic output price depend both on future expected monetary disturbances (represented by m_s) *and* future expected real disturbances (represented by u_s). However, the flex-price *real* exchange rate $\tilde{q}_t$, defined as $\tilde{e}_t - \tilde{p}_t$, is given by

$$\text{(A13)} \quad \tilde{q}_t = -\omega\int_t^\infty \exp[\omega(t-s)]u_s ds.$$

The real exchange rate thus depends exclusively on current and anticipated future real shocks; it is not influenced by monetary factors. Expression (A13) reflects the real-monetary dichotomy that characterizes the flex-price exchange-rate model.

2. STICKY PRICES AND THE BARRO-GROSSMAN ADJUSTMENT RULE

To introduce sticky prices and the Barro-Grossman adjustment scheme, assume that the domestic output price is a predetermined variable and replace the goods-market equilibrium condition (A3) with the equation

$$\text{(A14)} \quad \dot{p}_t = \pi[\phi(e_t - p_t + u_t) - \alpha\sigma(\dot{e}_t - \dot{p}_t)] + \dot{\tilde{p}}_t,$$

where $\tilde{p}_t$ again represents the flex-price equilibrium output price derived in the previous section. Equilibrium e_t and p_t in the present model satisfy (A2) and (A14), while $\tilde{e}_t$ and $\tilde{p}_t$ satisfy (A2) and (A3). It follows that under the Barro-Grossman price dynamics, the exchange rate and output price obey the equations

$$\text{(A15)} \quad \dot{e}'_t = [(1-\alpha)/\lambda]e'_t + (\alpha/\lambda)p'_t,$$

(A16) $$\dot{p}'_t = \pi[\phi(e'_t - p'_t) - \alpha\sigma(\dot{e}'_t - \dot{p}'_t)],$$

where $e'_t \equiv e_t - \tilde{e}_t$ and $p'_t \equiv p_t - \tilde{p}_t$. Equations (A15) and (A16), when combined, yield

(A17) $$\dot{p}'_t = \frac{[\lambda\pi\phi - \alpha(1-\alpha)\pi\sigma]}{\lambda(1-\pi\alpha\sigma)} e'_t - \frac{(\lambda\pi\phi + \alpha^2\pi\sigma)}{\lambda(1-\pi\alpha\sigma)} p'_t.$$

Together (A15) and (A17) describe an autonomous differential equation system in e'_t and p'_t.

The characteristic roots of that system, η_1 and η_2, are given by

$$\eta_1, \eta_2 = \frac{-[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]}{2\lambda(1-\pi\alpha\sigma)} \pm \left\{\frac{[\lambda\pi\phi + \pi\alpha\sigma - (1-\alpha)]^2}{4\lambda^2(1-\pi\alpha\sigma)^2} + \frac{\pi\phi}{\lambda(1-\pi\alpha\sigma)}\right\}^{1/2}.$$

Provided that the condition $1-\pi\alpha\sigma > 0$ is met, $\eta_1 > 0$ and $\eta_2 < 0$, as required for saddlepath stability when one of the two endogenous variables is predetermined. By imposing the requirement that the economy be on the stable saddlepath, we obtain the rational-expectations solution

(A18) $$e_t = \frac{-(p_0 - \tilde{p}_0)\alpha}{[(1-\alpha) - \lambda\eta_2]} \exp(\eta_2 t) + \tilde{e}_t,$$

(A19) $$p_t = (p_0 - \tilde{p}_0)\exp(\eta_2 t) + \tilde{p}_t.$$

According to (A18) and (A19), e_t and p_t converge to their flex-price values $\tilde{e}_t$ and $\tilde{p}_t$ at a rate given by $|\eta_2|$. At any point, the deviation of actual e_t or p_t from its flex-price value is proportional to the initial discrepancy between the predetermined initial output price p_0 and its flex-price level $\tilde{p}_0$ given by (A10). If there is no disequilibrium initially, a divergence between p_0 and $\tilde{p}_0$ can arise only as the result of previously unanticipated information arriving at time $t=0$.

3. STICKY PRICES AND THE MUSSA ADJUSTMENT RULE

The Mussa model results from replacing (A14) by the price-adjustment rule

(A20) $$\dot{p}_t = \theta[\phi(e_t - p_t + u_t) - \alpha\sigma(\dot{e}_t - \dot{p}_t)] + \dot{\bar{p}}_t,$$

where $\bar{p}_t$ is defined by the condition

(A21) $$\phi(e_t - \bar{p}_t + u_t) - \alpha\sigma(\dot{e}_t - \dot{p}_t) = 0.$$

If we differentiate (A21) and substitute the result into (A20), we obtain

(A22) $$\dot{p}_t = \theta[\phi(e_t - p_t + u_t) - \alpha\sigma(\dot{e}_t - \dot{p}_t)] + \dot{e}_t + \dot{u}_t - (1/\omega)(\ddot{e}_t - \ddot{p}_t)$$

(recall that $\omega \equiv \phi/\sigma\alpha$). Using the definition of the real exchange rate, $q_t \equiv e_t - p_t$, we may rewrite (A22) as a single, second-order differential equation in q_t,

(A23) $$\ddot{q}_t - \omega(1-\theta\alpha\sigma)\dot{q}_t - \omega\theta\phi q_t = \omega\theta\phi u_t + \omega\dot{u}_t.$$

Even though the level of q_t depends on the initial sticky nominal price of domestic goods, the adjustment law for q_t involves only real factors.

The model consisting of (A2) and (A22) will be solved in three steps. First, (A23) will be solved to obtain the rational-expectations path of the real exchange rate q_t. Second, equation (A2), rewritten as

$$\dot{e}_t = (1/\lambda)e_t - (\alpha/\lambda)q_t - m_t/\lambda, \tag{A24}$$

will be used in conjunction with the solution for q_t to obtain the path of e_t and the initial values e_0 and q_0. Third, the identity $p_t \equiv e_t - q_t$ will be used to derive the path of the domestic output price.

The Laplace transform, applied to (A23), yields the equation

$$L(q_t) = \frac{\dot{q}_0 + (l-\omega+\theta\phi)q_0 + \omega(\theta\phi+l)L(u_t)}{(l+\theta\phi)(l-\omega)}. \tag{A25}$$

Equation (A25) is based on the normalization $u_0=0$. After partial-fraction expansion, (A25) becomes

$$L(q_t) = -\left(\frac{\dot{q}_0-\omega q_0}{\omega+\theta\phi}\right)\left[\frac{1}{l+\theta\phi}\right] + \left[\left(\frac{\dot{q}_0+\theta\phi q_0}{\omega+\theta\phi}\right) + \omega L(u_t)\right]\left[\frac{1}{l-\omega}\right].$$

The Laplace transform theorem now imples that

$$\begin{aligned} q_t &= -\left(\frac{\dot{q}_0-\omega q_0}{\omega+\theta\phi}\right)\exp(-\theta\phi t) + \left(\frac{\dot{q}_0+\theta\phi q_0}{\omega+\theta\phi}\right)\exp(\omega t) \\ &\qquad + \omega\int_0^t \exp[\omega(t-s)]u_s ds \\ &= -\left(\frac{\dot{q}_0-\omega q_0}{\omega+\theta\phi}\right)\exp(-\theta\phi t) + \left[\left(\frac{\dot{q}_0+\theta\phi q_0}{\omega+\theta\phi}\right)\right. \\ &\qquad \left. + \omega\int_0^\infty \exp(-\omega s)u_s ds\right]\exp(\omega t) - \int_t^\infty \exp[\omega(t-s)]u_s ds. \end{aligned} \tag{A26}$$

The saddlepath assumption requires that the coefficient of $\exp(\omega t)$ in (A26) be zero. Thus, $\dot{q}_0$ and q_0 must satisfy the relation

$$\dot{q}_0 - \omega q_0 = -(\omega+\theta\phi)[q_0 + \omega\int_0^\infty \exp(-\omega s)u_s ds]. \tag{A27}$$

Combining (A13), (A26), and (A27), we find that

$$q_t = (q_0 - \tilde{q}_0)\exp(-\theta\phi t) + \tilde{q}_t. \tag{A28}$$

The effect of Mussa's price-adjustment mechanism is to drive the real exchange rate toward its flex-price level at a rate given by $\theta\phi$.

To solve for e_t, we write (A24) in terms of deviations from flex-price equilibrium values,

$$\dot{e}'_t = (1/\lambda)e'_t - (\alpha/\lambda)q'_t, \tag{A29}$$

where $e'_t \equiv e_t - \tilde{e}_t$ as before and $q'_t \equiv q_t - \tilde{q}_t$. Differentiation of (A28) gives

$$\dot{q}'_t = -\theta\phi q'_t. \tag{A30}$$

The characteristic roots of the autonomous system described by (A29) and (A30) are clearly $1/\lambda$ and $-\theta\phi$. Imposition of the saddlepath assumption leads to the exchange rate solution

$$e_t = \frac{(q_0 - \tilde{q}_0)\alpha}{(1+\lambda\theta\phi)} \exp(-\theta\phi t) + \tilde{e}_t. \tag{A31}$$

The initial values e_0 and q_0 may be recovered from (A31), given p_0. Substituting $e_0 - p_0$ for q_0 and setting $t=0$ gives

$$e_0 = \frac{-(p_0 - \tilde{p}_0)\alpha}{[(1-\alpha)+\lambda\theta\phi]} + \tilde{e}_0, \tag{A32}$$

$$q_0 = \frac{-(p_0 - \tilde{p}_0)(1+\lambda\theta\phi)}{[(1-\alpha)+\lambda\theta\phi]} + \tilde{q}_0. \tag{A33}$$

Thus, (A28) may be written as

$$q_t = \frac{-(p_0 - \tilde{p}_0)(1+\lambda\theta\phi)}{[(1-\alpha)+\lambda\theta\phi]} \exp(-\theta\phi t) + \tilde{q}_t, \tag{A34}$$

while (A31) takes the form

$$e_t = \frac{-(p_0 - \tilde{p}_0)\alpha}{[(1-\alpha)+\lambda\theta\phi]} \exp(-\theta\phi t) + \tilde{e}_t. \tag{A35}$$

The path of the domestic output price p_t is readily derived from (A34) and (A35). It is given by

$$p_t = (p_0 - \tilde{p}_0) \exp(-\theta\phi t) + \tilde{p}_t. \tag{A36}$$

In the wake of an unanticipated disturbance, the sticky price of domestic goods converges to its flex-price value at a rate given by $\theta\phi$.

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