Foundations of International Macroeconomics

Notation Guide

This notation guide is taken from *Foundations of International Macroeconomics*, by Maurice Obstfeld and Kenneth Rogoff (©MIT Press, September 1996)

Maurice Obstfeld
University of California at Berkeley

Kenneth Rogoff
Princeton Unviversity

1. Introduction

This notation guide and symbol glossary provides a brief summary of the book's basic notational conventions. Symbol usage is generally covered by the self-contained discussion within each chapter, but for convenience we also include here a listing of major symbol conventions that recur throughout the book.

2. Label Conventions

A number of our basic conventions for labeling variables can best be explained using consumption as an example.

2.1. One-Good, Representative-Agent Models

c^i: Consumption of agent i
c: Average per capita consumption.
C: Total aggregate consumption.

However, in representative-agent models, we typically normalize the population size to 1 and use C to denote both aggregate as well as individual consumption. (Nuance: In Chapters 4 and 10, C is a representative agent's consumption of a utility-weighted basket of different goods.)

c: The lowercase sans serif font refers to log C (or, in Chapters 7 and 10, an approximate log deviation of C from an initial baseline path).

2.2. Subscripts

C_t: Date t consumption (or C_s, for date s consumption)

Subscripts alongside t or s denote a type of good. For example:

$C_{\mathrm{N},t}$: Consumption of nontradables on date t.

(T is used for traded goods, H for home goods, F for foreign goods)

2.3. Superscripts

Superscripts other than i or j refer to a type of agent, for example:

$c^{\mathrm{O},i}$: Consumption of agent i, a member of the old generation.
c^{v}: Per capita consumption of agents in generation v.
C^{*}: Foreign consumption in a two-country model (where C is home consumption).
C^{n}: Aggregate consumption of country n.

Other examples include P for private agent, Y for member of young generation.

Superscripts may also be given special meanings in different chapters:

r^{A}: Autarky interest rate.
r^{C}: Consumption-based real interest rate.
r^{τ} : World interest rate in the presence of a tax on capital inflows.

Other examples include W for world, D for domestic.

2.4. Parentheses

It is sometimes more convenient to avoid subscript and superscript clutter by using an alternative convention:

$c_t^i(z)$ is agent i's consumption of good z at date t.

In Chapter 5, s denotes one of several possible randomly occurring states of nature:

$c_t^i(s)$ is the date t consumption of agent i in state s.

2.5. An Example

$c_{\mathrm{N},t}^i(s)$: Home agent i's date-t consumption of nontraded good N in state of nature s.

2.6. Bars, Primes, and Hats

Bars over a variable generally denote its steady-state or long-run constant value, for example:

$\bar{C}$: Steady-state aggregate consumption.

Exception: Overbars and underbars are used in combination to denote range of a variable; for example: $\overline{z}$ and $\underline{z}$ are upper and lower bounds for z.

Primes denote first deriviative when attached to a function of a single variable:

$C'(x) = \mathrm{d}C/\mathrm{d}x$.

However, a prime attached to a variable as opposed to a function denotes a specific value of that variable. Thus:

$\bar{M}'$ is a specific value of the steady-state money supply, $\bar{M}$.

Logarithmic changes and logarithmic approximations near steady-state values may be denoted by "hats", for example:

$\hat{C} = \dfrac{C - \bar{C}}{\bar{C}}$ is the percent deviation of variable C from steady state.
$\hat{\bar{C}} = \dfrac{\bar{C}' - \bar{C}}{\bar{C}}$ is the percent deviation of variable C's new steady state from its initial steady state.

> Exception: In Chapter 10 and in the stochastic growth model of Chapter 7, we use logarithmic changes so extensively that the hat notation (especially hatted barred variables) would become tedious. There we use c in place of $\hat{C}$, and $\bar{\mathsf{c}}$ in place of $\hat{\bar{C}}$.

3. Operators

Most operators are written as Roman letters.
$\mathrm{E}\{X\}$ = expectation of X.
$\mathrm{E}_t\{X_{t+1}\}$ = conditional (on date t information) expectation of X_{t+1}.
$\mathrm{Var}\{X\}$ = variance of X.
$\mathrm{Var}_t\{X_{t+1}\}$ = conditional (on date t information) variance of X_{t+1}.
$\mathrm{Std}\{X\}$ = standard deviation of X.
$\mathrm{Std}_t\{X_{t+1}\}$ = conditional (on date t information) standard deviation of X_{t+1}.
$\mathrm{Cov}\{X, Y\}$ = covariance of X and Y.
$\mathrm{Cov}_t\{X_{t+1}, Y_{t+1}\}$ = conditional (on date t information) covariance of X_{t+1} and Y_{t+1}.
$\mathrm{Corr}\{X, Y\}$ = correlation coefficient of X and Y.
$\mathrm{Corr}_t\{X_{t+1}, Y_{t+1}\}$ = conditional (on date t information) correlation of X_{t+1} and Y_{t+1}.
d = differential operator, $\mathrm{d}f(X)/\mathrm{d}X = f'(X)$.

Other operators include:
Lag operator, $L : LX_t = X_{t-1}$.
Lead (or forward) operator, $L^{-1} : L^{-1}X_t = X_{t+1}$. (In linear stochastic models, $L^{-1}E_tX_s = E_tX_{s+1}$).
First difference operator: $\triangle X_t = X_t - X_{t-1} = (1 - L)\, X_t$.
Matrix transposition operator: $\mathcal{T}$ defined so that for any matrix $[\mathsf{a}_{ij}]$, $[\mathsf{a}_{ij}]^{\mathcal{T}} = [\mathsf{a}_{ji}]$.

4. Timing of Asset Stocks and Interest Rates

The capital stock accumulated by the end of period t, and available for production in period $t+1$, is denoted K_{t+1}. Our timing convention for riskless bonds (private bonds and government debt) is the same. Correspondingly, r_{t+1} is the net real return on a riskless bond between periods t and $t+1$. These timing conventions are standard for discrete-time growth models; they are the main ones we employ. However, in Chapter 5, we follow standard finance notation and denote V_t as the end-of-period t market value of an asset that next yields dividends in period $t+1$. This timing convention for prices of claims is also used on occasion elsewhere.

In the special case of money, we denote by M_t the stock of nominal balances accumulated during period t. This notation, which we also use for other durable goods in section 2.4, is logical because we assume that durable goods begin to yield a service flow immediately upon purchase.

5. Sundry Conventions

Partial derivatives are generally written as, say, $F_L(K,L)$ rather $\partial F(K,L)/\partial L$, though we use the latter notation when it is clearer.

6. Major Variable and Parameter Definitions

Generally, the discussion of parameter usage within each chapter or set of related chapters is self-contained. Below, we list some of the recurring usages. Departures from the these conventions are clearly discussed in the context.

6.1. Uppercase English Alphabet

A: Productivity shift variable. $A(z)$ is relative productivity function in Chapter 4. Superscript A denotes autarky.
B: Net foreign assets: B^{G} denotes net government assets, B^{P} net private assets.
C: Aggregate consumption (equals per capita consumption when population is normalized to equal 1). In Chapters 4 and 10, index of real consumption.
D: Government debt ($= -B^{\mathrm{G}}$). Foreign debt ($= -B$) in Chapter 6. Stock of consumer durables in Chapter 2. Superscript D denotes "defaulting state" in Chapter 6 and indicates domestic interest rate in Chapter 7.

E: Level of Harrod-neutral technological change. $\mathrm{E}\{\cdot\}$ is the expectations operator.
$\mathbf{E}$ is eigenvector matrix in Supplement C to Chapter 2.
F: Production function $F(\cdot,\cdot)$. When it appears as super- or subscript, F indicates "foreign".
G: Government spending. $G(\cdot,\cdot)$ is a production function.
H: Human capital. When it appears as super- or subscript, H indicates "home."
I: Investment.
J: The function $J(\cdot)$ is the value function in dynamic programming.
K: Capital stock.
L: Labor supply. $\bar{L}$ denotes total time endowment. Gross lending in Chapter 6.
M: Money supply.
N: Total population, or total size of a generation. Superscript N denotes "nontraded."
O: Superscript O denotes "old" generation.
P: Price level.
Q: Financial wealth (total of securitized assets).
R: Market discount factor. ($R_{t,s}$ is the market discount factor between periods t and $s \geq t$.)
S: National saving. S^G is government savings, S^P private saving.
T: Aggregate taxes paid to government. Terminal time period. Superscript T denotes "traded."
U: Lifetime utility.
V: Market value of asset.
W: Beginning-of-period wealth, including the present value of wage income.
X: Generic variable.
Y: Gross domestic product. Endowment income. Superscript Y denotes "young."
Z: Net output $Y - G - I$ in Chapter 2. Nominal expenditure.

6.2. Lowercase English Alphabet

a: Generic constant.
b: Per capita net foreign assets. $b_{()}$ is a speculative bubble term in Chapter 8.
c: Per capita consumption.
d: Per capita government debt. d is the differential operator.
e: Harrod-neutral productivity growth-rate parameter. Eigenvector component.

f: $f(\cdot)$ is intensive form of a linear-homogeneous production function, $f(k) \equiv F(K/L, 1)$.
g: Output growth rate. Per capita government spending.
h: Per capita human capital. Length of time interval.
i : Investment per capita. Individual index variable. Nominal interest rate.
j: Individual index variable.
k: Per *worker* capital stock. In Chapter 8, k is (log) exchange-rate fundamentals.
l : Index variable.
m: Per capita nominal money holdings.
n: Population growth rate.
p: Relative price of nontraded goods. $p(s)$ is price of state s Arrow-Debreu security. Relative price of durable goods in Chapter 2.
q: Tobin's q. In Chapter 9, q denotes (log) real exchange rate.
r: Real interest rate. Rental price of capital. Rate of return on an asset.
s: Index variable for time, states of nature. Per capita saving.
t: Index variable for time.
u: Period utility function of individual is $u(\cdot, \ldots, \cdot)$.
v: Period derived utility from real balances, $v(\cdot)$.
w: Wage.
x: Generic weight variable.
y: Per capita GDP or endowment.
z: Index variable for commodities. Shock to output in Chapter 9.

6.3. Uppercase Greek Alphabet

Δ : First-difference operator.
Π : Profits. Product operator.
Σ : Summation operator.
Ω : Linear-homogeneous real consumption indexes are denoted by $\Omega(\cdot, \ldots, \cdot)$.
Ψ : Matrix of vector autoregression parameters in Chapter 2.

6.4. Lowercase Greek Alphabet

α: Capital's share in Cobb-Douglas production function.
β: Discount factor in individual intertemporal maximization problems.
γ: Generic weight parameter in utility function.

δ: Rate of subjective discount. Depreciation rate of capital. In Chapter 9, elasticity of aggregate demand with respect to the real exchange rate.
ϵ: Random shock.
ε: Inverse of consumption elasticity of demand for real balances.
ζ: In Chapter 1, import demand elasticity with respect to interest rate.
η: In Chapter 6, share of income creditors can destroy in event of default. In Chapter 8, semi-elasticity of money demand with respect to nominal interest rate (or inflation rate in the Cagan model).
θ: Elasticity of intratemporal substitution across different consumer goods. In Chapter 7, productivity parameter in production function for new capital-good varieties.
ϑ: Composite parameter equal to $1-(1+r)^{\sigma}\beta^{\sigma}$ in Chapter 2.
ι: Implicit rental cost (user cost) of consumer durable.
κ: Proportional transport cost in Chapter 4. Parameter for labor disutility in Chapter 10.
λ: Lagrange multiplier. Costate variable in Hamiltonian.
μ: Generic share parameter. Money-supply growth rate. Alternative Lagrange multiplier in Chapter 6.
ν: Alternative Lagrange multiplier.
ξ: Generic variable.
π: Inflation rate. As $\pi(s)$ in Chapter 5, denotes probability of state of nature s. In Chapter 6, $\pi(A)$ denotes probability density function of future investment productivity.
ρ: Coefficient of relative risk aversion. Autoregressive parameter.
σ: Elasticity of intertemporal substitution.
ς: Share of government spending in output in Chapter 2.
τ: Lump-sum per capita taxes or tax rate.
υ: Standard deviation of fundamentals in Chapter 8.
ϕ: Income elasticity of money demand. Production function parameter.
φ: Generic parameter.
χ: Installation cost paramenter in q model, Chapter 2. Utility-of-money weight in Chapter 10.
ψ: Vector autoregression parameters in Chapter 2. ψ_L is labor's share of GNP in Chapter 4. Alternative Lagrange multiplier in Chapter 6. Sensitivity of inflation to excess demand in Dornbusch model of Chapter 9.

ω: Global output shock in Chapter 6. Undetermined coefficient in Chapters 7 and 8. Eigenvalue.

6.5. Other

$\mathcal{A}$: $\mathcal{A}\left[u\left(c_t\right), U_{t+1}\right]$ denotes Koopmans's (1960) aggregator function.
$\mathcal{E}$: Nominal exchange rate (home price of foreign currency).
$\mathcal{F}$: forward exchange rate (home price of foreign currency).
$\mathcal{L}$: Lagrangian.
$\mathfrak{L}$: Loss function of monetary authorities in Chapter 9.
ℓ: Labor supply per worker in Chapter 10.
$\Re$: Repayment of debt in Chapter 6.
$\mathfrak{R}$: Reserve loss in speculative attack on a target zone, Chapter 8.
$\mathcal{S}$: Set of states of nature.
$\int$: Integral operator.