9 Nominal Price Rigidities: Empirical Facts and Basic Economy Models

$$\mathsf{i}_{t+1} = \mathsf{i}^* + \mathsf{e}_{t+1} - \mathsf{e}_t. \tag{1}$$

$$\mathsf{m}_t - \mathsf{p}_t = -\eta \mathsf{i}_{t+1} + \phi \mathsf{y}_t, \tag{2}$$

$$\mathsf{y}_t^d = \bar{\mathsf{y}} + \delta(\mathsf{e}_t + \mathsf{p}^* - \mathsf{p}_t - \bar{\mathsf{q}}), \qquad \delta > 0. \tag{3}$$

$$\mathsf{q} \equiv \mathsf{e} + \mathsf{p}^* - \mathsf{p}, \tag{4}$$

$$\mathsf{p}_{t+1} - \mathsf{p}_t = \psi(\mathsf{y}_t^d - \bar{\mathsf{y}}) + (\tilde{\mathsf{p}}_{t+1} - \tilde{\mathsf{p}}_t), \tag{5}$$

$$\tilde{\mathsf{p}}_t \equiv \mathsf{e}_t + \mathsf{p}_t^* - \bar{\mathsf{q}}_t$$

$$\tilde{\mathsf{p}}_{t+1} - \tilde{\mathsf{p}}_t = (\mathsf{e}_{t+1} + \mathsf{p}_{t+1}^* - \bar{\mathsf{q}}_{t+1}) - (\mathsf{e}_t + \mathsf{p}_t^* - \bar{\mathsf{q}}_t).$$

$$\mathsf{p}_{t+1} - \mathsf{p}_t = \psi(\mathsf{y}_t^d - \bar{\mathsf{y}}) + \mathsf{e}_{t+1} - \mathsf{e}_t \tag{6}$$

$$\Delta \mathsf{q}_{t+1} = \mathsf{q}_{t+1} - \mathsf{q}_t = -\psi\delta(\mathsf{q}_t - \bar{\mathsf{q}}) \tag{7}$$

$$\mathsf{m}_t - \mathsf{e}_t + \mathsf{q}_t = -\eta(\mathsf{e}_{t+1} - \mathsf{e}_t) + \phi\delta(\mathsf{q}_t - \bar{\mathsf{q}}) \tag{8}$$

$$\Delta \mathsf{e}_{t+1} = \mathsf{e}_{t+1} - \mathsf{e}_t = \frac{\mathsf{e}_t}{\eta} - \frac{(1-\phi\delta)\mathsf{q}_t}{\eta} - \left(\frac{\phi\delta\bar{\mathsf{q}} + \mathsf{m}_t}{\eta}\right) \tag{9}$$

$$\bar{e} = \bar{m} + \bar{q}, \tag{10}$$

$$\bar{p}' - \bar{p} = \bar{e}' - \bar{e} = \bar{m}' - \bar{m},$$

$$p_0 = \bar{m}, \tag{11}$$

$$q_0 = e_0 - \bar{m}. \tag{12}$$

$$m_t - p_t = -\eta i_{t+1} + \phi y_t.$$

$$q_s - \bar{q} = (1 - \psi\delta)^{s-t}(q_t - \bar{q}), \qquad s \geq t. \quad (13)$$

$$e_t - \bar{q} = \frac{\eta}{1+\eta}(e_{t+1} - \bar{q}) + \frac{1 - \phi\delta}{1+\eta}(q_t - \bar{q}) + \frac{m_t}{1+\eta}.$$

$$\mathsf{e}_t - \bar{\mathsf{q}} = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathsf{m}_s$$

$$+ \frac{1-\phi\delta}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} (\mathsf{q}_s - \bar{\mathsf{q}}) \tag{14}$$

$$\lim_{T\to\infty} \left(\frac{\eta}{1+\eta} \right)^{T} \mathsf{e}_{t+T} = 0.$$

$$\mathsf{e}_t - \bar{\mathsf{q}} = \bar{\mathsf{m}} + \frac{1-\phi\delta}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} (\mathsf{q}_s - \bar{\mathsf{q}}) \,. \tag{15}$$

$$\mathsf{e}_t - \bar{\mathsf{q}} = \bar{\mathsf{m}}$$

$$+ \frac{1 - \phi\delta}{1 + \eta}(\mathsf{q}_t - \bar{\mathsf{q}}) \sum_{s=t}^{\infty} (1 - \psi\delta)^{s-t} \left(\frac{\eta}{1 + \eta}\right)^{s-t}$$

$$\mathsf{e}_t = \bar{\mathsf{m}} + \bar{\mathsf{q}} + \frac{1 - \phi\delta}{1 + \psi\delta\eta}(\mathsf{q}_t - \bar{\mathsf{q}}). \tag{16}$$

$$\mathsf{q}_0 = \bar{\mathsf{q}} + \frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta}(\bar{\mathsf{m}}' - \bar{\mathsf{m}}).$$

$$\mathsf{e}_0 = \bar{\mathsf{m}} + \bar{\mathsf{q}} + \frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta}(\bar{\mathsf{m}}' - \bar{\mathsf{m}}). \tag{17}$$

$$\mathsf{e}_t = \bar{\mathsf{m}}' + \bar{\mathsf{q}} + (1 - \psi\delta)^t \left[\frac{1 - \phi\delta}{\phi\delta + \psi\delta\eta} (\bar{\mathsf{m}}' - \bar{\mathsf{m}}) \right].$$

$$\mathsf{e}_t - \bar{\mathsf{q}} = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathsf{m}_s$$

$$+ \frac{1 - \phi\delta}{1 + \psi\delta\eta} (\mathsf{q}_t - \bar{\mathsf{q}})$$

$$\mathsf{e}_t - \mathsf{e}_t^{flex} = \frac{1 - \phi\delta}{1 + \psi\delta\eta} (\mathsf{q}_t - \bar{\mathsf{q}}), \qquad (18)$$

$$\mathsf{e}_t^{flex} \equiv \bar{\mathsf{q}} + \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathsf{m}_s = \bar{\mathsf{q}} + \mathsf{p}_t^{flex} \quad (19)$$

$$\mathsf{q}_0 = \mathsf{e}_0 - \mathsf{p}_0^{flex},$$

$$\mathsf{e}_0 - \mathsf{e}_0^{flex} = \frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta} [(\mathsf{e}_0^{flex})' - \mathsf{e}_0^{flex}]. \quad (20)$$

$$\mathsf{e}_t - (\mathsf{e}_t^{flex})' = (1 - \psi\delta)^t [\mathsf{e}_0 - (\mathsf{e}_0^{flex})']. \quad (21)$$

$$\mathsf{m}_t = \bar{\mathsf{m}} + \mu t$$

$$\mathsf{m}_t = \bar{\mathsf{m}} + \mu' t, \quad \forall t \geq 0.$$

$$(\mathsf{e}_0^{flex})' - \mathsf{e}_0^{flex} = \eta(\mu' - \mu), \tag{22}$$

$$\mathsf{i}_{t+1} = \mathsf{i}^* + \mathsf{e}_{t+1} - \mathsf{e}_t = \mathsf{i}^* + (\mathsf{e}_{t+1}^{flex})' - (\mathsf{e}_t^{flex})' - \psi\delta(1 - \psi\delta)^t[\mathsf{e}_0 - (\mathsf{e}_0^{flex})']$$

$$\mathsf{i}_{t+1} - (\mathsf{i}^* + \mu) = (\mu' - \mu)$$

$$- \psi\delta(1 - \psi\delta)^t[\mathsf{e}_0 - (\mathsf{e}_0^{flex})'] \qquad (23)$$

$$(\mu' - \mu) - \psi\delta\eta \left(\frac{1 - \phi\delta}{\phi\delta + \psi\delta\eta}\right)(\mu' - \mu)$$

$$= \phi\delta \left(\frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta}\right)(\mu' - \mu)$$

$$\mathsf{i}_{t+1} - (\mathsf{p}_{t+1} - \mathsf{p}_t) = (\mathsf{e}_{t+1} - \mathsf{e}_t) - (\mathsf{p}_{t+1} - \mathsf{p}_t)$$

$$= -\psi\delta(\mathsf{e}_t + \mathsf{p}^* - \mathsf{p}_t - \bar{\mathsf{q}}) \qquad (24)$$

$$[\mathsf{i}_{t+1} - (\mathsf{p}_{t+1} - \mathsf{p}_t)] - [\mathsf{i}^*_{t+1} - (\mathsf{p}^*_{t+1} - \mathsf{p}^*_t)]$$

$$= -\psi\delta(\mathsf{e}_t - \mathsf{p}_t + \mathsf{p}^*_t - \bar{\mathsf{q}}) \tag{25}$$

$$\mathsf{q}_t = a_0 + \rho\mathsf{q}_{t-1} + \epsilon_t,$$

$$\log(IP_{1935}/IP_{1929}) = 2.45 + 0.49\log(WPI_{1935}/WPI_{1929}),$$

$$(0.21)\ (0.23)$$

$$\mathsf{m}_t - \mathsf{p}_t = -\eta\mathsf{i}_{t+1} + \phi\mathsf{y}_t + \epsilon_t,$$

$$m_t - \bar{m} = \Phi(e_t - \bar{e}),$$

$$y_t = \bar{y} - (w_t - p_t) - z_t, \tag{26}$$

$$w_t = \mathrm{E}_{t-1} p_t. \tag{27}$$

$$\pi_t \equiv p_t - p_{t-1}. \tag{28}$$

$$\mathfrak{L}_t = (\mathsf{y}_t - \tilde{\mathsf{y}})^2 + \chi \pi_t^2. \tag{29}$$

$$\tilde{\mathsf{y}} - \bar{\mathsf{y}} = k > 0 \tag{30}$$

$$\mathfrak{L}_t = (\pi_t - \pi_t^{\mathrm{e}} - z_t - k)^2 + \chi \pi_t^2, \tag{31}$$

$$\frac{\mathrm{d}\mathfrak{L}_t}{\mathrm{d}\pi_t} = \underbrace{2(\pi_t - \pi_t^{\mathrm{e}} - z_t - k)}_{\substack{\text{(minus) marginal benefit of} \\ \text{higher inflation}}} + \underbrace{2\chi\pi_t}_{\substack{\text{marginal cost of} \\ \text{higher inflation}}} = 0, \tag{32}$$

$$\pi_t = \frac{k + \pi_t^{\mathrm{e}} + z_t}{1 + \chi}. \tag{33}$$

$$\pi_t^{\mathrm{e}} = \mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\left\{\frac{k + \pi_t^{\mathrm{e}} + z_t}{1 + \chi}\right\},$$

$$\pi_t^{\mathrm{e}} = \frac{k}{\chi}. \tag{34}$$

$$\pi_t = \frac{k}{\chi} + \frac{z_t}{1 + \chi}. \tag{35}$$

$$\pi_t = \frac{z_t}{1+\chi}, \tag{36}$$

$$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \mathfrak{L}_s \right\}, \tag{37}$$

$$\pi_t^{\mathrm{e}} = \begin{cases} 0 & \text{if } \pi_{t-s} = \pi_{t-s}^{\mathrm{e}}, \quad \forall s > 0 \\ k/\chi & \text{otherwise.} \end{cases} \tag{38}$$

$$\mathfrak{L}_t = (\pi_t - k)^2 + \chi \pi_t^2,$$

$$\left(\frac{k}{1+\chi} - k\right)^2 + \chi\left(\frac{k}{1+\chi}\right)^2 = \frac{\chi}{1+\chi}k^2$$

$$\pi_t^{\mathrm{e}} = \begin{cases} 0 & \text{if } \pi_{t-s} = \pi_{t-s}^{\mathrm{e}} + z_{t-s}/(1+\chi), \quad \forall s > 0 \\ k/\chi & \text{otherwise.} \end{cases} \tag{39}$$

$$\mathfrak{L}_t^{\mathrm{CB}} = (\pi_t - \pi_t^{\mathrm{e}} - z_t - k)^2 + \chi^{\mathrm{CB}}\pi_t^2$$

$$\pi_t^{\mathrm{e}} = \frac{k}{\chi^{\mathrm{CB}}}, \tag{40}$$

$$\pi_t = \frac{k}{\chi^{\mathrm{CB}}} + \frac{z_t}{1 + \chi^{\mathrm{CB}}}. \tag{41}$$

$$\mathfrak{L}_t^{\mathrm{CB}} = (\pi_t - \pi_t^{\mathrm{e}} - z_t - k)^2 + \chi\pi_t^2 + 2\omega\pi_t \tag{42}$$

$$\frac{\mathrm{d}\mathfrak{L}_t^{\mathrm{CB}}}{\mathrm{d}\pi_t} = 2(\pi_t - \pi_t^{\mathrm{e}} - z_t - k + \chi\pi_t + \omega) = 0.$$

$$\pi_t^{\mathrm{e}} = \frac{k - \omega}{\chi}, \tag{43}$$

$$\pi_t = \frac{k - \omega}{\chi} + \frac{z_t}{1 + \chi}, \tag{44}$$

$$\mathfrak{L}_t^{\mathrm{L}} = -(\pi_t - \pi_t^{\mathrm{e}} - k) + \frac{\chi^{\mathrm{L}}}{2}\pi_t^2, \tag{45}$$

$$\mathfrak{L}_t^{\mathrm{C}} = \pi_t^2. \tag{46}$$

$$\pi_t^{\mathrm{e}} = \pi_t = \frac{1}{\chi^{\mathrm{L}}}. \tag{47}$$

$$\pi_t^{\mathrm{e}} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{\chi^{\mathrm{L}}} = \frac{1}{2\chi^{\mathrm{L}}}. \tag{48}$$

$$\pi_{1973-94} = \underset{(1.57)}{8.30} - \underset{(2.35)}{6.02}CBI, \qquad R^2 = 0.30$$

$$\mathfrak{L}_t = (\mathsf{y}_t - \tilde{\mathsf{y}})^2 + \chi\pi_t^2 + C(\pi_t), \tag{49}$$

$$\mathsf{y} = \bar{\mathsf{y}} + (\pi - \pi^{\mathrm{e}}) - z; \tag{50}$$

$$\pi = \frac{k + \pi^{\mathrm{e}} + z}{1 + \chi} \tag{51}$$

$$\mathsf{y} = \bar{\mathsf{y}} + \frac{k - \chi\pi^{\mathrm{e}} - \chi z}{1 + \chi}, \tag{52}$$

$$\mathfrak{L}^{\mathrm{FLEX}} = \frac{\chi}{1 + \chi}(k + \pi^{\mathrm{e}} + z)^2. \tag{53}$$

$$\mathfrak{L}^{\mathrm{FIX}} = (k + z + \pi^{\mathrm{e}})^2 > \mathfrak{L}^{\mathrm{FLEX}}. \tag{54}$$

$$\bar{z} = \sqrt{\bar{c}(1+\chi)} - k - \pi^{\mathrm{e}}, \tag{55}$$

$$\underline{z} = -\sqrt{\underline{c}(1+\chi)} - k - \pi^{\mathrm{e}}. \tag{56}$$

$$\begin{aligned} \mathrm{E}\pi &= \mathrm{E}\{\pi \mid z < \underline{z}\}\Pr(z < \underline{z}) \\ &\quad + \mathrm{E}\{\pi \mid z > \bar{z}\}\Pr(z > \bar{z}), \end{aligned} \tag{57}$$

$$\mathrm{E}\pi = \frac{1}{1+\chi}\left[\left(1 - \frac{\bar{z} - \underline{z}}{2Z}\right)(k + \pi^{\mathrm{e}}) - \frac{\bar{z}^2 - \underline{z}^2}{4Z}\right] \tag{58}$$

$$\pi^{\mathrm{e}} = \mathrm{E}\pi,$$

$$\frac{\mathrm{dE}\pi}{\mathrm{d}\pi^{\mathrm{e}}} = \begin{cases} \dfrac{1}{1+\chi} & (\text{for } \underline{z} > -Z) \\ \dfrac{1}{1+\chi}\left[\dfrac{1}{2} + \dfrac{1}{2Z}\left(k + \pi^{\mathrm{e}}\right)\right] & (\text{for } \underline{z} = -Z,\ \ \bar{z} > -Z) \\ \dfrac{1}{1+\chi} & (\text{for } \bar{z} = -Z). \end{cases}$$

$$\begin{aligned} \mathsf{y}_t - \bar{\mathsf{y}} = a_1[\Delta \mathsf{m}_t - \mathrm{E}_{t-1}\Delta \mathsf{m}_t] \\ + a_2[\Delta \mathsf{m}_t^* - \mathrm{E}_{t-1}\Delta \mathsf{m}_t^*] + \epsilon_t \end{aligned}$$

$$y_t^* - \bar{y} = a_1[\Delta m_t^* - E_{t-1}\Delta m_t^*] + a_2[\Delta m_t - E_{t-1}\Delta m_t] + \epsilon_t^*$$

$$\mathfrak{L}_t = (y_t - \bar{y})^2 + \chi(\Delta m_t)^2,$$

$$\mathfrak{L}_t^* = (y_t^* - \bar{y})^2 + \chi(\Delta m_t^*)^2.$$

$$\frac{\partial \mathfrak{L}_t}{\partial m_t} = 2a_1(y_t - \bar{y}) + 2\chi\Delta m_t = 0.$$

$$\frac{\partial \mathfrak{L}_t^*}{\partial m_t^*} = 2a_1(y_t^* - \bar{y}) + 2\chi\Delta m_t^* = 0.$$

$$x\mathfrak{L}_t + (1-x)\mathfrak{L}_t^*.$$

$$x\frac{\partial \mathfrak{L}_t}{\partial \mathsf{m}_t} + (1-x)\frac{\partial \mathfrak{L}_t^*}{\partial \mathsf{m}_t} = 0,$$

$$x\frac{\partial \mathfrak{L}_t}{\partial \mathsf{m}_t^*} + (1-x)\frac{\partial \mathfrak{L}_t^*}{\partial \mathsf{m}_t^*} = 0,$$

Table 9.1 Comparing Exchange-Rate and Stock-Price-Index Volatility, January 1981–August 1994 (standard deviation of month-to-month log changes)

Dollar/DM	Dollar/yen	S&P 500	Commerzbank	Nikkei
2.9	2.8	3.4	5.7	5.9