

8 Money and Exchange Rates under Flexible Prices

$$m_t^d - p_t = -\eta E_t \{p_{t+1} - p_t\}, \quad (1)$$

$$\frac{M_t^d}{P_t} = L(Y_t, i_{t+1}). \quad (2)$$

$$1 + i_{t+1} = (1 + r_{t+1}) \frac{P_{t+1}}{P_t}. \quad (3)$$

$$m_t^d = m_t.$$

$$m_t - p_t = -\eta E_t \{p_{t+1} - p_t\}. \quad (4)$$

$$m_t - p_t = -\eta (p_{t+1} - p_t). \quad (5)$$

$$p_t = \frac{1}{1 + \eta} m_t + \frac{\eta}{1 + \eta} p_{t+1}, \quad (6)$$

$$p_{t+1} = \frac{1}{1 + \eta} m_{t+1} + \frac{\eta}{1 + \eta} p_{t+2};$$

$$p_t = \frac{1}{1 + \eta} \left(m_t + \frac{\eta}{1 + \eta} m_{t+1} \right) + \left(\frac{\eta}{1 + \eta} \right)^2 p_{t+2}.$$

$$p_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} m_s + \lim_{T \rightarrow \infty} \left(\frac{\eta}{1 + \eta} \right)^T p_{t+T} \quad (7)$$

$$\lim_{T \rightarrow \infty} \left(\frac{\eta}{1 + \eta} \right)^T \mathbf{p}_{t+T} = 0. \quad (8)$$

$$\mathbf{p}_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} \mathbf{m}_s. \quad (9)$$

$$\frac{1}{1 + \eta} \left[1 + \frac{\eta}{1 + \eta} + \left(\frac{\eta}{1 + \eta} \right)^2 + \dots \right]$$

$$= \frac{1}{1 + \eta} \left(\frac{1}{1 - \frac{\eta}{1 + \eta}} \right) = 1$$

$$m_t = \bar{m} + \mu t.$$

$$p_t = m_t + \eta \mu. \tag{10}$$

$$m_t = \begin{cases} \bar{m} & t < T, \\ \bar{m}' & t \geq T. \end{cases}$$

$$p_t = \begin{cases} \bar{m} + \left(\frac{\eta}{1+\eta}\right)^{T-t} (\bar{m}' - \bar{m}), & t < T, \\ p_t = \bar{m}', & t \geq T. \end{cases}$$

$$p_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} m_s + b_0 \left(\frac{1 + \eta}{\eta} \right)^t, \quad (11)$$

$$b_0 = p_0 - \frac{1}{1 + \eta} \sum_{s=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^s m_s.$$

$$p_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} E_t\{m_s\}, \quad (12)$$

$$m_t = \rho m_{t-1} + \epsilon_t, \quad 0 \leq \rho \leq 1, \quad (13)$$

$$\begin{aligned}
p_t &= \frac{m_t}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta\rho}{1 + \eta} \right)^{s-t} \\
&= \left(\frac{m_t}{1 + \eta} \right) \frac{1}{1 - \frac{\eta\rho}{1+\eta}} = \frac{m_t}{1 + \eta - \eta\rho}
\end{aligned}
\tag{14}$$

$$m_t - p_t = -\eta\dot{p}_t, \tag{15}$$

$$p_t = \frac{1}{\eta} \int_t^{\infty} \exp[-(s - t)/\eta] m_s ds + b_0 \exp(t/\eta) \tag{16}$$

$$m_t - p_t = -\frac{\eta}{h}(p_{t+h} - p_t). \quad (17)$$

$$p_t = \frac{1}{1 + \eta/h} \sum_{s=t}^{\infty} \left(\frac{\eta/h}{1 + \eta/h} \right)^{(s-t)/h} m_s + b_0 \left(\frac{1 + \eta/h}{\eta/h} \right)^{t/h}$$

$$p_t = \frac{1}{h + \eta} \sum_{s=t}^{\infty} \left(1 + \frac{h}{\eta} \right)^{-(s-t)/h} m_s h + b_0 \left(1 + \frac{h}{\eta} \right)^{t/h}$$

$$\begin{aligned}
p_t &= \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} m_s ds = m_t \\
&+ \int_t^\infty e^{-(s-t)/\eta} \dot{m}_s ds = m_t + \eta\mu
\end{aligned}
\tag{18}$$

$$p_t = m_t + \eta\mu + b_0 e^{t/\eta},
\tag{19}$$

$$\text{Seignorage} = \frac{M_t - M_{t-1}}{P_t}
\tag{20}$$

$$\text{Seignorage} = \frac{M_t - M_{t-1}}{M_t} \cdot \frac{M_t}{P_t} \quad (21)$$

$$\frac{M_t}{P_t} = \left(\frac{P_{t+1}}{P_t} \right)^{-\eta}.$$

$$1 + \mu = \frac{M_t}{M_{t-1}} = \frac{P_t}{P_{t-1}},$$

$$\text{Seignorage} = \frac{\mu}{1 + \mu} \cdot (1 + \mu)^{-\eta} = \mu(1 + \mu)^{-\eta-1} \quad (22)$$

$$(1 + \mu)^{-\eta-1} - \mu(\eta + 1)(1 + \mu)^{-\eta-2} = 0,$$

$$\mu^{\text{MAX}} = \frac{1}{\eta}. \quad (23)$$

$$m_t - p_t = -\eta i_{t+1} + \phi y_t, \quad (24)$$

$$P_t = \mathcal{E}_t P_t^* \quad (25)$$

$$p_t = e_t + p_t^*. \quad (26)$$

$$1 + i_{t+1} = (1 + i_{t+1}^*) E_t \left\{ \frac{\varepsilon_{t+1}}{\varepsilon_t} \right\}. \quad (27)$$

$$i_{t+1} = i_{t+1}^* + E_t e_{t+1} - e_t \quad (28)$$

$$(m_t - \phi y_t + \eta i_{t+1}^* - p_t^*) - e_t = -\eta (E_t e_{t+1} - e_t) \quad (29)$$

$$\mathbf{e}_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} \mathbf{E}_t \{ \mathbf{m}_s - \phi \mathbf{y}_s + \eta \mathbf{i}_{s+1}^* - \mathbf{p}_s^* \}$$

(30)

$$\mathbf{m}_t - \mathbf{m}_{t-1} = \rho (\mathbf{m}_{t-1} - \mathbf{m}_{t-2}) + \epsilon_t, \quad 0 \leq \rho \leq 1$$

(31)

$$\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} \mathbf{E}_t \{ \mathbf{m}_{s+1} - \mathbf{m}_s \}$$

(32)

$$\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t = \frac{\rho}{1 + \eta - \eta\rho} (\mathbf{m}_t - \mathbf{m}_{t-1}).$$

$$e_t = m_t + \frac{\eta\rho}{1 + \eta - \eta\rho}(m_t - m_{t-1}).$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left(C_s, \frac{M_s}{P_s} \right), \quad (33)$$

$$P_t = \varepsilon_t P^*,$$

$$\alpha \log C + (1 - \alpha) \log(\bar{L} - L_t),$$

$$\bar{L} - L_t = \bar{L} \left(\frac{M_t/P_t}{C_t} \right)^\varepsilon,$$

$$\alpha \log C + (1 - \alpha) \log(\bar{L} - L_t)$$

$$= [\alpha - \varepsilon(1 - \alpha)] \log C_t + \varepsilon(1 - \alpha) \log \frac{M_t}{P_t}$$

$$B_{t+1} + \frac{M_t}{P_t} = (1 + r)B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - T_t, \quad (34)$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left[-B_{s+1} - \frac{M_s}{P_s} + (1+r)B_s + \frac{M_{s-1}}{P_s} + Y_s - T_s, \frac{M_s}{P_s} \right]$$

$$u_C \left(C_t, \frac{M_t}{P_t} \right) = (1+r)\beta u_C \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right), \quad (35)$$

$$\begin{aligned} \frac{1}{P_t} u_C \left(C_t, \frac{M_t}{P_t} \right) &= \frac{1}{P_t} u_{M/P} \left(C_t, \frac{M_t}{P_t} \right) \\ &+ \frac{1}{P_{t+1}} \beta u_C \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right) \end{aligned} \quad (36)$$

$$\frac{u_{M/P} \left(C_t, \frac{M_t}{P_t} \right)}{u_C \left(C_t, \frac{M_t}{P_t} \right)} = 1 - \frac{P_t/P_{t+1}}{1+r} = \frac{i_{t+1}}{1+i_{t+1}} \quad (37)$$

$$\begin{aligned} \frac{B_{t+2}}{1+r} + \frac{M_{t+1}}{P_{t+1}(1+r)} &= B_{t+1} \\ + \frac{M_t}{P_{t+1}(1+r)} + \frac{Y_{t+1} - C_{t+1} - T_{t+1}}{1+r} \end{aligned}$$

$$\begin{aligned}
& \frac{B_{t+2}}{1+r} + \frac{M_{t+1}}{P_{t+1}(1+r)} \\
&= \left(B_{t+1} + \frac{M_t}{P_t} \right) - \frac{M_t}{P_t} \left[1 - \frac{P_t}{P_{t+1}(1+r)} \right] \\
& \quad + \frac{Y_{t+1} - C_{t+1} - T_{t+1}}{1+r}
\end{aligned}$$

$$1 - \frac{P_t}{P_{t+1}(1+r)} = \frac{i_{t+1}}{1+i_{t+1}}$$

$$\begin{aligned}
& \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[C_s + \frac{i_{s+1}}{1+i_{s+1}} \left(\frac{M_s}{P_s} \right) \right] \\
& = (1+r)B_t + \frac{M_{t-1}}{P_t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - T_s)
\end{aligned} \tag{38}$$

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T \left(B_{t+T+1} + \frac{M_{t+T}}{P_{t+T}} \right) = 0$$

$$u \left(C, \frac{M}{P} \right) = \frac{[C^\gamma (M/P)^{1-\gamma}]^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \tag{39}$$

$$\frac{M_t^d}{P_t} = \left(\frac{1 - \gamma}{\gamma} \right) \left(1 + \frac{1}{i_{t+1}} \right) C_t. \quad (40)$$

$$u\left(C, \frac{M}{P}\right) = \frac{\left\{ \left[\gamma^{\frac{1}{\theta}} C^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} \left(\frac{M}{P} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \right\}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$

$$P_t^C \equiv \left[\gamma + (1 - \gamma) \left(\frac{i_{t+1}}{1 + i_{t+1}} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

$$1 + r_{t+1}^c = (1 + r) \frac{P_t^c}{P_{t+1}^c}.$$

$$\frac{M_t^d}{P_t} = \left(\frac{1 - \gamma}{\gamma} \right) \left(1 + \frac{1}{i_{t+1}} \right)^\theta C_t.$$

$$\Omega \left(C, \frac{M}{P} \right) = \left[\gamma^{\frac{1}{\theta}} C^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} \left(\frac{M}{P} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

$$\frac{Z_t}{P_t^c} = \frac{(1 + r)B_t + \frac{M_{t-1}}{P_t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - T_s)}{P_t^c \sum_{s=t}^{\infty} \left[(1 + r)^{s-t} \left(\frac{P_t^c}{P_s^c} \right) \right]^{\sigma-1} \beta^{\sigma(s-t)}}.$$

$$C_t = \frac{\gamma}{(P_t^c)^{-\theta}} \cdot \frac{(1+r)B_t + \frac{M_{t-1}}{P_t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s)}{P_t^c \sum_{s=t}^{\infty} \left[\prod_{v=t+1}^s (1+r_v^c)\right]^{\sigma-1} \beta^{\sigma(s-t)}}.$$

$$C_t = \gamma(1-\beta) \left[(1+r)B_t + \frac{M_{t-1}}{P_t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s) \right]$$

$$G_t = T_t + \frac{M_t - M_{t-1}}{P_t}, \quad (41)$$

$$B_{t+1} = (1+r)B_t + Y_t - G_t - C_t. \quad (42)$$

$$-T_t = \frac{M_t - M_{t-1}}{P_t}. \quad (43)$$

$$\bar{C} = r B_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s) \quad (44)$$

$$u \left(C, \frac{M}{P} \right) = \log C + v \left(\frac{M}{P} \right), \quad (45)$$

$$\frac{v' \left(\frac{M_t}{P_t} \right)}{1/\bar{C}} = 1 - \frac{P_t/P_{t+1}}{1+r}$$

$$\frac{M_{t+1}}{P_{t+1}} \left(\frac{\beta}{1+\mu} \right) = \frac{M_t}{P_t} \left[1 - \bar{C} v' \left(\frac{M_t}{P_t} \right) \right] \quad (46)$$

$$\frac{\bar{M}}{\bar{P}} = \bar{C} \left(\frac{1}{1 - \frac{\beta}{1+\mu}} \right) .$$

$$\frac{1}{P_t} \left(\frac{1}{C_t} \right) = \frac{1}{P_t} v' \left(\frac{M_t}{P_t} \right) + \frac{1}{P_{t+1}} \left(\frac{\beta}{C_{t+1}} \right) \quad (47)$$

$$\frac{1}{P_t} \left(\frac{1}{C_t} \right) = \frac{1}{P_t} v' \left(\frac{M_t}{P_t} \right) + \frac{\beta}{P_{t+1}} v' \left(\frac{M_{t+1}}{P_{t+1}} \right) + \frac{1}{P_{t+2}} \left(\frac{\beta^2}{C_{t+2}} \right)$$

$$\frac{1}{P_t} \left(\frac{1}{\bar{C}} \right) = \sum_{s=t}^{t+T-1} \beta^{s-t} \frac{1}{P_s} v' \left(\frac{\bar{M}}{P_s} \right) + \frac{1}{P_{t+T}} \left(\frac{\beta^T}{\bar{C}} \right) \quad (48)$$

$$\frac{1}{P_t} \left(\frac{1}{\bar{C}} \right) = \sum_{s=t}^{\infty} \beta^{s-t} \frac{1}{P_s} v' \left(\frac{\bar{M}}{P_s} \right), \quad (49)$$

$$\lim_{T \rightarrow \infty} \frac{1}{P_{t+T}} \cdot \frac{\beta^T}{\bar{C}} = 0 \quad (50)$$

$$\frac{1}{P_{t+1}} = \frac{1}{\beta P_t} - \frac{\bar{C} v'(\bar{M}/P_t)}{\beta P_t}. \quad (51)$$

$$\lim_{T \rightarrow \infty} \frac{1}{P_{t+T}} \cdot \frac{\beta^T}{\bar{C}} > 0$$

$$\frac{1}{P_t} \left(\frac{1}{\bar{C}} \right) > \sum_{s=t}^{\infty} \frac{1}{P_s} \beta^{s-t} v' \left(\frac{\bar{M}}{P_s} \right).$$

$$\lim_{M/P \rightarrow 0} \frac{M}{P} v' \left(\frac{M}{P} \right) = 0, \quad (52)$$

$$\frac{1}{\bar{C}} = v' \left(\frac{\bar{M}}{P_{T-1}} \right) \quad (53)$$

$$\lim_{M/P \rightarrow 0} \frac{M}{P} v' \left(\frac{M}{P} \right) > 0. \quad (54)$$

$$m_t - p_t = -\eta i_{t+1} + \phi y_t + \epsilon_t$$

$$\begin{aligned}
& (\mathbf{m}_t - \phi \mathbf{y}_t + \eta \mathbf{i}_{t+1}^* - \mathbf{p}_t^*) - \mathbf{e}_t \\
& = -\eta (\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t) + \epsilon_t
\end{aligned} \tag{55}$$

$$\mathbf{e}_t = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^{s-t} \mathbf{E}_t \{ \mathbf{m}_s - \phi \mathbf{y}_s + \eta \mathbf{i}_{s+1}^* - \mathbf{p}_s^* - \epsilon_s \} \tag{56}$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \tag{57}$$

$$B_{t+1} + \frac{M_t}{P_t} = (1 + r) B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - T_t$$

$$M_{t-1} \geq P_t C_t. \quad (58)$$

$$M_{t-1} = P_t C_t$$

$$B_{t+1} = (1 + r)B_t + Y_t - T_t - \frac{P_{t+1}}{P_t} C_{t+1}. \quad (59)$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left\{ \frac{P_{s-1}}{P_s} [(1 + r)B_{s-1} - B_s + Y_{s-1} - T_{s-1}] \right\}$$

$$\frac{P_{s-1}}{P_s} u'(C_s) = (1+r) \frac{P_s}{P_{s+1}} \beta u'(C_{s+1}).$$

$$\frac{u'(C_s)}{1+i_s} = (1+r) \beta \frac{u'(C_{s+1})}{1+i_{s+1}} \quad (60)$$

$$\frac{M_{t-1}}{P_t} = C_t.$$

$$M_t \geq P_t C_t.$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left\{ u(C_s) + v \left[\frac{M_s}{P_s} + g \left(\frac{\mathcal{E}_s M_{F,s}}{P_s} \right) \right] \right\} \quad (61)$$

$$g \left(\frac{\mathcal{E} M_F}{P} \right) = a_0 \left(\frac{\mathcal{E} M_F}{P} \right) - \frac{a_1}{2} \left(\frac{\mathcal{E} M_F}{P} \right)^2, \quad (62)$$

$$\begin{aligned} B_{t+1} + \frac{M_t}{P_t} + \frac{\mathcal{E}_t M_{F,t}}{P_t} &= (1+r)B_t \\ &+ \frac{M_{t-1}}{P_t} + \frac{\mathcal{E}_t M_{F,t-1}}{P_t} + Y_t - C_t - T_t \end{aligned} \quad (63)$$

$$u'(C_t) = u'(C_{t+1}), \quad (64)$$

$$\frac{1}{P_t} u'(C_t) = \frac{1}{P_t} v' \left[\frac{M_t}{P_t} + g \left(\frac{M_{F,t}}{P_t^*} \right) \right] + \frac{1}{P_{t+1}} \beta u'(C_{t+1}) \quad (65)$$

$$\frac{1}{P_t^*} u'(C_t) = \frac{1}{P_t^*} v' \left[\frac{M_t}{P_t} + g \left(\frac{M_{F,t}}{P_t^*} \right) \right] g' \left(\frac{M_{F,t}}{P_t^*} \right) + \frac{1}{P_{t+1}^*} \beta u'(C_{t+1}) \quad (66)$$

$$g' \left(\frac{M_{F,t}}{P_t^*} \right) = \frac{1 - \beta \frac{P_t^*}{P_{t+1}^*}}{1 - \beta \frac{P_t}{P_{t+1}}}.$$

$$\frac{M_{F,t}}{P_t^*} = \frac{1}{a_1} \left(a_0 - \frac{1 - \beta \frac{P_t^*}{P_{t+1}^*}}{1 - \beta \frac{P_t}{P_{t+1}}} \right) \quad (67)$$

$$m_t - e_t = -\eta(\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t). \quad (68)$$

$$m_t - \bar{e} = -\eta(\bar{e} - \bar{e})$$

$$m_t = \bar{m} = \bar{e}.$$

$$m_t - \bar{e}_t = -\eta\mu$$

$$u \left(C, \frac{M}{P} \right) = \log C + \log \left(\frac{M}{P} \right),$$

$$\frac{M_t}{\mathcal{E}_t} = \bar{C} \left[\frac{1+r}{1+r - (\mathcal{E}_t/\mathcal{E}_{t+1})} \right], \quad (69)$$

$$\frac{M_t}{\mathcal{E}_t} = (\bar{Y} + rB_t - \bar{G}) \left[\frac{(1+\mu)(1+r)}{(1+\mu)(1+r) - 1} \right].$$

$$m_t - e_t = -\eta \dot{e}_t. \quad (70)$$

$$\bar{m} = \bar{e}. \quad (71)$$

$$M_t = B_{H,t} + \bar{\varepsilon} B_{F,t}. \quad (72)$$

$$\frac{\dot{B}_H}{B_H} = \dot{\mathbf{b}}_H = \mu, \quad (73)$$

$$\bar{\varepsilon} \dot{B}_F = -\dot{B}_H. \quad (74)$$

$$\tilde{\mathbf{e}}_t = \mathbf{b}_{H,t} + \eta\mu, \quad (75)$$

$$\mathbf{b}_{H,t} = \mathbf{b}_{H,0} + \mu t,$$

$$\bar{\mathbf{e}} = \mathbf{b}_{H,0} + \mu T + \eta\mu.$$

$$T = \frac{\bar{\mathbf{e}} - \mathbf{b}_{H,0} - \eta\mu}{\mu}. \quad (76)$$

$$T = \frac{\log(B_{H,0} + B_{F,0}) - \mathbf{b}_{H,0} - \eta\mu}{\mu}. \quad (77)$$

$$\tilde{\mathbf{e}}_t = \log B_{H,t} + \eta\mu + b_T e^{(t-T)/\eta},$$

$$T = \frac{\log(B_{H,0} + B_{F,0}) - \mathbf{b}_{H,0} - \eta\mu - b_T}{\mu},$$

$$\mathbf{p}_t - \mathbf{p}_t^* = \mathbf{m}_t - \mathbf{m}_t^* - \phi(\mathbf{y}_t - \mathbf{y}_t^*) + \eta(\mathbf{i}_{t+1} - \mathbf{i}_{t+1}^*).$$

$$\mathbf{e}_t = \mathbf{m}_t - \mathbf{m}_t^* - \phi(\mathbf{y}_t - \mathbf{y}_t^*) + \eta(\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t). \quad (78)$$

$$\mathbf{e}_t = \mathbf{m}_t - \mathbf{m}_t^* - \phi(\mathbf{y}_t - \mathbf{y}_t^*) + \eta(\mathbf{E}_t \mathbf{e}_{t+1} - \mathbf{e}_t)$$

$$\mathbf{e}_t = \mathbf{m}_t - \mathbf{m}_t^* - \phi(\mathbf{y}_t - \mathbf{y}_t^*) + \frac{\eta}{h} \mathbf{E}_t d\mathbf{e}_{t+h} \quad (79)$$

$$\mathbf{e}_t = \frac{1}{h + \eta} \sum_{s=t}^{\infty} \left(1 + \frac{h}{\eta}\right)^{-(s-t)/h} \mathbf{E}_t \{\mathbf{k}_s\} h, \quad (80)$$

$$d\mathbf{k}_{t+h} = h^{1/2} v dz_{t+h}, \quad (81)$$

$$e = G(\mathbf{k}),$$

$$\begin{aligned} G(\mathbf{k}_t) &= \mathbf{k}_t + \frac{\eta}{h} \mathbf{E}_t dG(\mathbf{k}_{t+h}) \\ &= \mathbf{k}_t + \frac{\eta}{h} \mathbf{E}_t \{G(\mathbf{k}_{t+h}) - G(\mathbf{k}_t)\}. \end{aligned} \quad (82)$$

$$\begin{aligned} &\mathbf{E}_t \{G(\mathbf{k}_{t+h}) - G(\mathbf{k}_t)\} \\ &\approx \mathbf{E}_t \left\{ G'(\mathbf{k}_t) d\mathbf{k}_{t+h} + \frac{1}{2} G''(\mathbf{k}_t) (d\mathbf{k}_{t+h})^2 \right\} \end{aligned} \quad (83)$$

$$E_t\{(dk_{t+h})^2\} = E_t\{hv^2(dz_{t+h})^2\} = hv^2.$$

$$E_t dG(k_{t+h}) \approx \frac{hv^2}{2} G''(k_t).$$

$$G(k) = k + \frac{\eta v^2}{2} G''(k) \tag{84}$$

$$G(k) = k + b_1 \exp(\lambda k) + b_2 \exp(-\lambda k),$$

$$\lambda = \sqrt{\frac{2}{\eta v^2}}.$$

$$G(k) = k - b[\exp(\lambda k) - \exp(-\lambda k)]. \quad (85)$$

$$\begin{aligned} E_t\{S(\bar{k} + dk) - S(\bar{k})\} &= S'(\bar{k})E_t\{dk \mid dk \leq 0\} \\ &+ \frac{1}{2}S''(\bar{k})E_t\{(dk)^2 \mid dk \leq 0\} \end{aligned} \quad (86)$$

$$S(\bar{k}) = \bar{k} + \frac{\eta}{h} S'(\bar{k}) E_t \{ dk \mid dk \leq 0 \} \\ + \frac{\eta v^2}{2} S''(\bar{k}). \quad (87)$$

$$S(k) = k + \frac{\eta v^2}{2} S''(k).$$

$$S(\bar{k}) = \lim_{k \rightarrow \bar{k}} S(k) = \bar{k} + \frac{\eta v^2}{2} S''(\bar{k}).$$

$$S(\bar{k}) = \bar{k} - b \left[\exp(\lambda \bar{k}) - \exp(-\lambda \bar{k}) \right] = \bar{e},$$

$$S'(\bar{k}) = 1 - \lambda b \left[\exp(\lambda \bar{k}) + \exp(-\lambda \bar{k}) \right] = 0.$$

$$S(\bar{k}) = \bar{k} + b_1 \exp(\lambda \bar{k}) = \bar{e}. \quad (88)$$

$$S(\bar{k}) = \bar{k} + b_1 \exp(\lambda \bar{k}) = \bar{k} - \mathfrak{R}, \quad (89)$$

$$S'(\bar{k}) = 1 + \lambda b_1 \exp(\lambda \bar{k}) = 0. \quad (90)$$

$$\mathcal{R}' = \frac{1}{\lambda}.$$

$$U_t^n = \mathbf{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s^n) + v \left(\frac{M_s^n}{P_s^n} \right) \right], \quad n = 1, \dots, N \quad (91)$$

$$P^n = \varepsilon^{nm} P^m, \quad (92)$$

$$u'(C_t) = \beta \mathbf{E}_t \{ (1 + r_{t+1}^m) u'(C_{t+1}) \}, \quad (93)$$

$$C_t^n = x^n Y_t^W \quad (94)$$

$$\frac{1}{P_t} u'(C_t) = (1 + i_{t+1}) \beta E_t \left\{ \frac{1}{P_{t+1}} u'(C_{t+1}) \right\}$$

$$u'(C_t) = \beta E_t \left\{ (1 + i_{t+1}) \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\}, \quad (95)$$

$$u'(C_t) = (1 + r_{t+1}) \beta E_t \{ u'(C_{t+1}) \}. \quad (96)$$

$$\begin{aligned}
& (1 + r_{t+1})\mathbf{E}_t \{u'(C_{t+1})\} \\
& = (1 + i_{t+1})\mathbf{E}_t \left\{ \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\}
\end{aligned}$$

$$1 + r_{t+1} = (1 + i_{t+1})\mathbf{E}_t \left\{ \frac{P_t}{P_{t+1}} \right\} \tag{97}$$

$$\begin{aligned}
& \mathbf{E}_t \left\{ \frac{P_t}{P_{t+1}} u'(C_{t+1}) \right\} \\
& = \mathbf{E}_t \left\{ \frac{P_t}{P_{t+1}} \right\} \mathbf{E}_t \{u'(C_{t+1})\}
\end{aligned}$$

$$\frac{1}{P_t} u'(C_t) = \frac{1}{P_t} v' \left(\frac{M_t}{P_t} \right) + \beta E_t \left\{ \frac{1}{P_{t+1}} u'(C_{t+1}) \right\} \quad (98)$$

$$\frac{v' \left(\frac{M_t}{P_t} \right)}{u'(C_t)} = \frac{i_{t+1}}{1 + i_{t+1}}.$$

$$u(C_t) + v \left(\frac{M_t}{P_t} \right) = \frac{C^{1-\rho}}{1-\rho} + \log \left(\frac{M_t}{P_t} \right)$$

$$1 = \frac{P_t (x Y_t^W)^\rho}{M_t} + \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{Y_t^W}{Y_{t+1}^W} \right)^\rho \right\} \quad (99)$$

$$\frac{M_{t+1}}{M_t} = (1 + \mu) \epsilon_{t+1}, \quad (100)$$

$$\frac{M_t}{P_t} = \omega (x Y_t^W)^\rho, \quad (101)$$

$$\omega = \frac{1 + \mu}{1 + \mu - \beta} = \frac{1}{1 - \frac{\beta}{1 + \mu}} > 0.$$

$$P_t = M_t \left(1 - \frac{\beta}{1 + \mu} \right) (x Y_t^W)^{-\rho}$$

$$\begin{aligned} m_t - p_t &= \rho y_t^W - (1/\bar{i}) E_t \{ p_{t+1} - p_t \} \\ &\quad - (\rho/\bar{i}) E_t \{ y_{t+1}^W - y_t^W \} \end{aligned}$$

$$p_t = -\rho y_t^W + \left(\frac{\bar{i}}{1 + \bar{i}} \right) \sum_{s=t}^{\infty} \left(\frac{1}{1 + \bar{i}} \right)^{s-t} E_t \{ m_s \},$$

$$U_t^n = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^n). \quad (102)$$

$$P_t^n = \frac{M_t^n}{Y_t^n} \quad (103)$$

$$1 + i_{t+1} = (1 + i_{t+1}^*) \frac{\mathcal{F}_t}{\mathcal{E}_t}. \quad (104)$$

$$\mathcal{F}_t = E_t \{ \mathcal{E}_{t+1} \} \quad (105)$$

$$\mathbf{E}_t \left\{ \frac{1}{\mathcal{E}_{t+1}} \right\} > \frac{1}{\mathbf{E}_t \{ \mathcal{E}_{t+1} \}}$$

$$\frac{1}{\mathcal{F}_t} = \mathbf{E}_t \left\{ \frac{1}{\mathcal{E}_{t+1}} \right\},$$

$$\mathbf{f}_t = \mathbf{E}_t \{ \mathbf{e}_{t+1} \} + \frac{1}{2} \text{Var}_t(\mathbf{e}_{t+1}). \quad (106)$$

$$\mathbf{E}_t \left\{ \frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right\} = 0. \quad (107)$$

$$\mathbf{E}_t \left\{ \frac{\frac{1}{\mathcal{F}_t} - \frac{1}{\boldsymbol{\varepsilon}_{t+1}}}{P_{t+1}^*} \right\} = 0 \quad (108)$$

$$\begin{aligned} \mathbf{f}_t = \mathbf{E}_t\{\mathbf{e}_{t+1}\} + \frac{1}{2} \text{Var}_t(\mathbf{e}_{t+1}) \\ - \text{Cov}_t(\mathbf{e}_{t+1}, \mathbf{p}_{t+1}). \end{aligned} \quad (109)$$

$$\mathbf{e}_{t+1} - \mathbf{e}_t = a_0 + a_1(\mathbf{f}_t - \mathbf{e}_t) + \boldsymbol{\epsilon}_t, \quad (110)$$

$$\text{plim}(a_1^{\text{OLS}}) = \frac{\text{Cov}(\mathbf{f}_t - \mathbf{e}_t, \mathbf{e}_{t+1} - \mathbf{e}_t)}{\text{Var}(\mathbf{f}_t - \mathbf{e}_t)}. \quad (111)$$

$$rp_t = f_t - E_t\{e_{t+1}\},$$

$$f_t - e_t = E_t\{e_{t+1}\} - e_t + rp_t. \quad (112)$$

$$e_{t+1} - E_t\{e_{t+1}\},$$

$$E_t\{(f_t - e_t)(e_{t+1} - E_t\{e_{t+1}\})\} = 0$$

$$\text{plim}(a_1^{\text{OLS}}) = \frac{\text{Cov} [\mathbf{f}_t - \mathbf{e}_t, \mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t]}{\text{Var}(\mathbf{f}_t - \mathbf{e}_t)} \quad (113)$$

$$\begin{aligned} & \text{Cov}[\mathbf{f}_t - \mathbf{e}_t, \mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t] \\ &= \text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t) + \text{Cov}[\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t, r p_t] \end{aligned}$$

$$\text{Cov}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t, r p_t) < 0.$$

$$\begin{aligned}
& \text{plim}(a_1^{\text{OLS}}) \{ \text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t) \\
& \quad + 2\text{Cov}[\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t, rp_t] + \text{Var}(rp_t) \} \\
& = \text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t) \\
& \quad + \text{Cov}[\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t, rp_t] \tag{114}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} [\text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t) \\
& \quad + \text{Var}(rp_t)] > \text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t)
\end{aligned}$$

$$\text{Var}(rp_t) > \text{Var}(\mathbf{E}_t\{\mathbf{e}_{t+1}\} - \mathbf{e}_t). \tag{115}$$

$$0 = \mathbf{E}_t \left\{ \left(r_{t+1}^n - r_{t+1}^m \right) \frac{u'(C_{t+1})}{u'(C_t)} \right\}, \quad (116)$$

$$\frac{(1 + i_{t+1})P_t}{P_{t+1}} - \frac{(1 + i_{t+1}^*)P_t^*}{P_{t+1}^*}.$$

$$\frac{(1 + i_{t+1})P_t}{\mathcal{F}_t} \left(\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right).$$

$$0 = \mathbf{E}_t \left\{ \left(\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) \frac{u'(C_{t+1})}{u'(C_t)} \right\}. \quad (117)$$

$$0 = \mathbf{E}_t \left\{ \left(\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) \left(\frac{C_t}{C_{t+1}} \right)^\rho \right\}. \quad (118)$$

$$\begin{aligned} \mathbf{f}_t - \mathbf{E}_t\{\mathbf{e}_{t+1}\} &= \frac{1}{2} \text{Var}_t(\mathbf{e}_{t+1}) \\ &\quad - \text{Cov}_t(\mathbf{e}_{t+1}, \mathbf{p}_{t+1}) - \rho \text{Cov}_t(\mathbf{e}_{t+1}, \mathbf{c}_{t+1}) \end{aligned} \quad (119)$$

$$0 = \mathbf{E}_t \left\{ \left(\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) \left(\frac{Y_t^W}{Y_{t+1}^W} \right)^\rho \right\}$$

$$\mathbf{f}_t = \mathbf{E}_t\{\mathbf{e}_{t+1}\} + \frac{1}{2} \text{Var}_t(\mathbf{e}_{t+1}) \\ - \text{Cov}_t(\mathbf{e}_{t+1}, \mathbf{p}_{t+1}) - \rho \text{Cov}_t(\mathbf{e}_{t+1}, \mathbf{y}_{t+1}^{\mathbf{W}})$$

$$\frac{1}{1 + i_{t+1}^*} (1 + i_{t+1}) / \mathcal{F}_t = 1 / \mathcal{E}_t$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s). \quad (120)$$

$$C = \left[\gamma^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (121)$$

$$p_H = \mathcal{E} p_H^*, \quad (122)$$

$$p_F = \mathcal{E} p_F^*. \quad (123)$$

$$M_{H,t-1} \geq p_{H,t} C_{H,t}, \quad (124)$$

$$M_{F,t-1} \geq p_{F,t}^* C_{F,t}, \quad (125)$$

$$M_{H,t-1}^* \geq p_{H,t} C_{H,t}^*$$

$$M_{F,t-1}^* \geq p_{F,t}^* C_{F,t}^*$$

$$P = \left[\gamma p_H^{1-\theta} + (1 - \gamma) p_F^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (126)$$

$$P^* = \left[\gamma (p_H^*)^{1-\theta} + (1 - \gamma) (p_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (127)$$

$$P = \mathcal{E} P^*.$$

$$\begin{aligned}
P_t B_{t+1} + M_{H,t} + \mathcal{E}_t M_{F,t} &= P_t (1 + r_t) B_t \\
+ M_{H,t-1} + \mathcal{E}_t M_{F,t-1} + p_{H,t} Y_t - p_{H,t} C_{H,t} & \quad (128) \\
- p_{F,t} C_{F,t} - P_t T_t &
\end{aligned}$$

$$\begin{aligned}
P_t^* B_{t+1}^* + M_{F,t}^* + \frac{M_{H,t}^*}{\mathcal{E}_t} & \\
= P_t^* (1 + r_t) B_t^* + M_{F,t-1}^* + \frac{M_{H,t-1}^*}{\mathcal{E}_t} & \\
+ p_{F,t}^* Y_t^* - p_{H,t}^* C_{H,t}^* - p_{F,t}^* C_{F,t}^* - P_t^* T_t^* &
\end{aligned}$$

$$T_t = \frac{M_{H,t} + M_{H,t}^* - M_{H,t-1} - M_{H,t-1}^*}{P_t},$$

$$T_t^* = \frac{M_{F,t} + M_{F,t}^* - M_{F,t-1} - M_{F,t-1}^*}{P_t^*}.$$

$$\frac{P_{s-1}}{P_s} u'(C_s) = (1 + r) \frac{P_s}{P_{s+1}} \beta u'(C_{s+1}), \quad (129)$$

$$C_{H,s} = \frac{\gamma}{1 - \gamma} \left(\frac{p_{F,s}}{p_{H,s}} \right)^\theta C_{F,s}. \quad (130)$$

$$\frac{p_H}{p_F} = \left[\frac{\gamma Y_F}{(1 - \gamma) Y_H} \right]^{1/\theta} . \quad (131)$$

$$r = \frac{1 - \beta}{\beta} .$$

$$\begin{aligned} P^g \text{Gold}^{cb} + \varepsilon B_F^{cb} + B_H^{cb} + \varepsilon M_F^{cb} \\ = M_H + RR + NW \end{aligned}$$

Table 8.1 Foreign Exchange Reserves and Monetary Base, September 1994

	Monetary Base (percent of GNP)	Reserves (percent of GNP)	Reserves/Base (percent)
Belgium	6.7	12.1	180
Denmark	8.6	8.1	94
Finland	11.2	10.4	93
France	4.6	4.6	100
Germany	9.9	6.2	63
Ireland	9.1	16.1	177
Italy	11.9	5.6	48
Mexico	3.9	4.7	120
Netherlands	10.0	13.6	136
Norway	6.3	18.7	297
Portugal	25.0	28.0	112
Spain	12.6	9.6	76
Sweden	13.0	12.1	93
United Kingdom	3.7	4.3	116

Source: Obstfeld and Rogoff (1995c).

Central Bank Balance Sheet

Assets

Liabilities

Net foreign-currency bonds

Monetary base

Net domestic-currency bonds

Net worth

Foreign money

Gold

Central Bank Balance Sheet

Assets

Liabilities

Foreign-currency bonds (+1)

Monetary base (+1)

Central Bank Balance Sheet

Assets

Liabilities

Home-currency bonds (−1) Monetary base (−1)

Central Bank Balance Sheet

Assets

Liabilities

Home-currency bonds (−1) Monetary base (—)
Foreign-currency bonds (+1)
