6 Imperfections in International Capital Markets

$$U_1 = \mathrm{E}u(C_2),$$

$$Y_2 = \bar{Y} + \epsilon,$$

$$\sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i) = 0. \tag{1}$$

$$C_2(\epsilon) = Y_2 - P(\epsilon) = Y_2 - \epsilon = \bar{Y}.$$

$$\sum_{i=1}^{N} \pi(\epsilon_i) Y_2 = \sum_{i=1}^{N} \pi(\epsilon_i) \bar{Y} + \sum_{i=1}^{N} \pi(\epsilon_i) \epsilon_i = \bar{Y}$$

$$P(\epsilon_i) \leq \eta(\bar{Y} + \epsilon_i). \tag{2}$$

$$\max_{C_2(\epsilon), P(\epsilon)} \sum_{i=1}^{N} \pi(\epsilon_i) u[C_2(\epsilon_i)]$$

$$C_2(\epsilon_i) = \bar{Y} + \epsilon_i - P(\epsilon_i). \tag{3}$$

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)]$$

$$- \sum_{i=1}^{N} \lambda(\epsilon_i)[P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i)]$$

$$+ \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i)$$

$$\pi(\epsilon) u'[C_2(\epsilon)] + \lambda(\epsilon) = \mu \pi(\epsilon), \tag{4}$$

$$\lambda(\epsilon)[\eta(\bar{Y} + \epsilon) - P(\epsilon)] = 0, \tag{5}$$

$$
\begin{aligned}
u'(\bar{Y} - P_0) - u'[C_2(\epsilon)] \\
&= u'(\bar{Y} - P_0) - u'[\bar{Y} + \epsilon - P(\epsilon)] \\
&= u'(\bar{Y} - P_0) - u'[(1-\eta)(\bar{Y} + \epsilon)] \\
&= \frac{\lambda(\epsilon)}{\pi(\epsilon)} \geq 0. \qquad (6)
\end{aligned}
$$

$$\bar{Y} - P_0 = (1-\eta)(\bar{Y} + e), \qquad (7)$$

$$P_0 + e = \eta(\bar{Y} + e). \qquad (8)$$

$$P(\epsilon) = \begin{cases} \eta\bar{Y} - (1-\eta)e + \epsilon = \eta(\bar{Y}+e) + (\epsilon - e), & \epsilon \in [\underline{\epsilon}, e) \\ \eta(\bar{Y}+\epsilon) = \eta(\bar{Y}+e) + \eta(\epsilon - e), & \epsilon \in [e, \bar{\epsilon}] \end{cases} \tag{9}$$

$$\int_{-\bar{\epsilon}}^{e} [\eta(\bar{Y}+e) + (\epsilon - e)] \frac{\mathrm{d}\epsilon}{2\bar{\epsilon}} + \int_{e}^{\bar{\epsilon}} [\eta(\bar{Y}+e) + \eta(\epsilon - e)] \frac{\mathrm{d}\epsilon}{2\bar{\epsilon}} = 0$$

$$e^2 + 2\bar{\epsilon}e + \left(\bar{\epsilon}^2 - \frac{4\eta\bar{\epsilon}\bar{Y}}{1-\eta}\right) = 0.$$

$$e = -\bar{\epsilon} + 2\sqrt{\frac{\eta\bar{\epsilon}\bar{Y}}{1-\eta}}. \tag{10}$$

$$U_1 = u(C_1) + \beta \mathrm{E} u(C_2), \qquad \beta < 1,$$

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \tag{11}$$

$$B_{s+1} = (1+r)B_s + \bar{Y} + \epsilon_s - C_s - P_s(\epsilon_s) \tag{12}$$

$$\sum_{i=1}^{N} \pi(\epsilon_i) P_s(\epsilon_i) = 0 \tag{13}$$

$$\text{Gain}(\epsilon_t) = u(\bar{Y} + \epsilon_t) - u(\bar{Y}). \tag{14}$$

$$\text{Cost} = \sum_{s=t+1}^{\infty} \beta^{s-t} u(\bar{Y}) - \sum_{s=t+1}^{\infty} \beta^{s-t} \mathrm{E}_t u(\bar{Y} + \epsilon_s)$$

$$\text{Cost} = \frac{\beta}{1-\beta}[u(\bar{Y}) - \mathrm{E}u(\bar{Y} + \epsilon)]. \tag{15}$$

$$\text{Gain}(\bar{\epsilon}) \leq \text{Cost},$$

$$u(\bar{Y} + \bar{\epsilon}) - u(\bar{Y}) \leq \frac{\beta}{1-\beta}[u(\bar{Y}) - \mathrm{E}u(\bar{Y} + \epsilon)]. \qquad (16)$$

$$Y_s = (1+g)^{s-t}\bar{Y} \exp[\epsilon_s - \frac{1}{2}\mathrm{Var}(\epsilon)],$$

$$u(C) = \frac{C^{1-\rho}}{1-\rho},$$

$$\beta \bar{U}_{t+1} = \frac{1}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t} (1+g)^{(1-\rho)(s-t)} \bar{Y}^{1-\rho}$$

$$= \frac{\bar{Y}^{1-\rho}}{1-\rho} \times \frac{\beta(1+g)^{1-\rho}}{1-\beta(1+g)^{1-\rho}}$$

$$\beta \mathrm{E}_t U_{t+1}^{\mathrm{A}} = \frac{1}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t} \mathrm{E}_t Y_s^{1-\rho}$$

$$= \frac{\bar{Y}^{1-\rho}}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t} (1+g)^{(1-\rho)(s-t)} \mathrm{E}_t \exp\left\{(1-\rho)\left[\epsilon_s - \frac{1}{2}\mathrm{Var}(\epsilon)\right]\right\}$$

$$= \frac{\bar{Y}^{1-\rho}}{1-\rho} \sum_{s=t+1}^{\infty} \beta^{s-t} (1+g)^{(1-\rho)(s-t)} \exp\left\{\frac{1}{2}[(1-\rho)^2 - (1-\rho)]\mathrm{Var}(\epsilon)\right\}$$

$$= \frac{\bar{Y}^{1-\rho}}{1-\rho} \times \frac{\beta(1+g)^{1-\rho}}{1-\beta(1+g)^{1-\rho}} \exp\left[-\frac{1}{2}\rho(1-\rho)\mathrm{Var}(\epsilon)\right] < \beta \bar{U}_{t+1}.$$

$$\lim_{Y_t \to \infty} [u(Y_t) - u(\bar{Y})] \leq \beta(\bar{U}_{t+1} - \mathrm{E}_t U^{\mathrm{A}}_{t+1}).$$

$$\frac{\lim_{\epsilon_t \to \infty} \exp\{(1-\rho)[\epsilon_t - \frac{1}{2}\mathrm{Var}(\epsilon)]\} - 1}{1-\rho}$$

$$\leq \frac{\beta(1+g)^{1-\rho}}{(1-\rho)[1-\beta(1+g)^{1-\rho}]} \left\{1 - \exp\left[-\frac{1}{2}\rho(1-\rho)\mathrm{Var}(\epsilon)\right]\right\}$$

$$1 \leq \beta(1+g)^{1-\rho} \exp\left[\frac{1}{2}\rho(\rho-1)\mathrm{Var}(\epsilon)\right].$$

$$u[(1+\kappa)\bar{Y}] - u(\bar{Y}) = \beta(\bar{U}_{t+1} - \mathrm{E}_t U^{\mathrm{A}}_{t+1}),$$

$$\kappa = \left[\frac{1 - \beta(1+g)^{1-\rho} \exp\left[\frac{1}{2}\rho(\rho-1)\mathrm{Var}(\epsilon)\right]}{1 - \beta(1+g)^{1-\rho}} \right]^{-\frac{1}{\rho-1}} - 1$$

$$C_s(\epsilon_s) = \bar{Y} + \epsilon_s - P_s(\epsilon_s) \tag{17}$$

$$\mathrm{Gain}(\epsilon_t) = u(\bar{Y} + \epsilon_t) - u[\bar{Y} + \epsilon_t - P(\epsilon_t)].$$

$$\mathrm{Cost} = \frac{\beta}{1-\beta} \left\{ \mathrm{E}u[\bar{Y} + \epsilon - P(\epsilon)] - \mathrm{E}u(\bar{Y} + \epsilon) \right\}.$$

$$
\begin{aligned}
&u(\bar{Y} + \epsilon_t) - u[\bar{Y} + \epsilon_t - P(\epsilon_t)] \\
&\quad \leq \frac{\beta}{1-\beta} \left\{ \mathrm{E}u[\bar{Y} + \epsilon - P(\epsilon)] - \mathrm{E}u(\bar{Y} + \epsilon) \right\} \\
&\quad = \frac{\beta}{1-\beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left\{ u[\bar{Y} + \epsilon_j - P(\epsilon_j)] - u(\bar{Y} + \epsilon_j) \right\}. \qquad (18)
\end{aligned}
$$

$$
\sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)]
$$

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\epsilon_i) u[\bar{Y} + \epsilon_i - P(\epsilon_i)]$$

$$- \sum_{i=1}^{N} \lambda(\epsilon_i) \Big(u(\bar{Y} + \epsilon_i) - u[\bar{Y} + \epsilon_i - P(\epsilon_i)]$$

$$- \frac{\beta}{1-\beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left\{ u[\bar{Y} + \epsilon_j - P(\epsilon_j)] - u(\bar{Y} + \epsilon_j) \right\} \Big)$$

$$+ \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i).$$

$$\left[\pi(\epsilon) + \lambda(\epsilon) + \frac{\beta \pi(\epsilon)}{1-\beta} \sum_{j=1}^{N} \lambda(\epsilon_j) \right] u'[C(\epsilon)] = \mu \pi(\epsilon)$$

(19)

$$\lambda(\epsilon)\Big(\frac{\beta}{1-\beta}\sum_{j=1}^{N}\pi(\epsilon_j)\left\{u[\bar{Y}+\epsilon_j-P(\epsilon_j)]-u(\bar{Y}+\epsilon_j)\right\}$$

$$-u(\bar{Y}+\epsilon)+u[\bar{Y}+\epsilon-P(\epsilon)]\Big)=0, \tag{20}$$

$$u'[C(\epsilon)]=\frac{\mu}{1+\frac{\beta}{1-\beta}\sum_{j=1}^{N}\lambda(\epsilon_j)}. \tag{21}$$

$$\frac{\mathrm{d}P(\epsilon)}{\mathrm{d}\epsilon}=\frac{u'[\bar{Y}+\epsilon-P(\epsilon)]-u'(\bar{Y}+\epsilon)}{u'[\bar{Y}+\epsilon-P(\epsilon)]}.$$

$$\mu = \left[1 + \frac{\beta}{1-\beta} \sum_{j=1}^{N} \lambda(\epsilon_j) \right] u'(\bar{Y} - P_0).$$

$$\left[1 + \frac{\beta}{1-\beta} \sum_{j=1}^{N} \lambda(\epsilon_j) \right] \left\{ u'(\bar{Y} - P_0) - u'[C(\epsilon)] \right\}$$

$$= \frac{\lambda(\epsilon) u'[C(\epsilon)]}{\pi(\epsilon)}$$

$$U_t^j = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^j) \right\}.$$

$$Y_t^j = \bar{Y} + \epsilon_t^j + \omega_t,$$

$$\sum_j \epsilon_t^j = 0. \tag{22}$$

$$C_t^j = \bar{Y} + \omega_t, \qquad \forall j, t.$$

$$\text{Gain}(\epsilon_t^j, \omega_t) = u(\bar{Y} + \epsilon_t^j + \omega_t) - u(\bar{Y} + \omega_t),$$

$$\text{Cost} = \mathrm{E}_t \sum_{s=t+1}^{\infty} \beta^{s-t}[u(\bar{Y} + \omega_s) - u(\bar{Y} + \epsilon_s^j + \omega_s)]$$

$$= \frac{\beta}{1-\beta}[\mathrm{E}u(\bar{Y} + \omega) - \mathrm{E}u(\bar{Y} + \epsilon^j + \omega)].$$

$$\text{Gain}(\bar{\epsilon}, \underline{\omega}) \quad \leq \quad \text{Cost}.$$

$$U_1 = u(C_1) + \beta u(C_2).$$

$$Y_2 = F(K_2).$$

$$K_2 = Y_1 + D_2 - C_1,$$

$$C_2 = F(K_2) + K_2 - \Re,$$

$$\Re = \min \{(1+r)D_2, \eta[F(K_2) + K_2]\}, \tag{23}$$

$$F'(K_2) = r$$

$$u'(C_1) = (1+r)\beta u'(C_2).$$

$$\Re \leq \eta[F(K_2) + K_2].$$

$$\max_{K_2} u(Y_1 + D_2 - K_2)$$
$$+ \beta u \left[F(K_2) + K_2 - \min\{(1+r)D_2, \eta[F(K_2) + K_2]\}\right] \quad (24)$$

$$1 + r > \eta(1 + \alpha). \quad (25)$$

$$C_1 + \frac{C_2}{1+\alpha} = Y_1 + \frac{(\alpha - r)}{1+\alpha} D_2. \tag{26}$$

$$\begin{aligned} C_1 &= \frac{1}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha} D_2\right], \\ C_2 &= \frac{(1+\alpha)\beta}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha} D_2\right] \end{aligned} \tag{27}$$

$$\begin{aligned} U^{\mathrm{N}} = (1+\beta)\log\left\{\frac{1}{1+\beta}\left[Y_1 + \frac{(\alpha - r)}{1+\alpha} D_2\right]\right\} \\ + \beta \log\left[(1+\alpha)\beta\right] \end{aligned}$$

$$C_1 + \frac{C_2}{(1-\eta)(1+\alpha)} = Y_1 + D_2. \tag{28}$$

$$\begin{aligned} C_1 &= \frac{1}{1+\beta}(Y_1 + D_2), \\ C_2 &= \frac{(1-\eta)(1+\alpha)\beta}{1+\beta}(Y_1 + D_2), \end{aligned} \tag{29}$$

$$U^{\mathrm{D}} = (1+\beta)\log\left[\frac{1}{1+\beta}(Y_1 + D_2)\right] + \beta\log[(1-\eta)(1+\alpha)\beta]$$

$$U^{\mathrm{D}} - U^{\mathrm{N}} = (1+\beta)\log\left[\frac{1+\dfrac{D_2}{Y_1}}{1+\dfrac{(\alpha-r)}{1+\alpha}\left(\dfrac{D_2}{Y_1}\right)}\right]$$

$$+\,\beta\log(1-\eta)$$

$$1 = \left[\frac{1+\dfrac{D_2}{Y_1}}{1+\dfrac{(\alpha-r)}{1+\alpha}\left(\dfrac{D_2}{Y_1}\right)}\right]^{1+\beta}(1-\eta)^{\beta}.$$

$$\bar{D} = \left[\frac{\left(\frac{1}{1-\eta}\right)^{\beta/(1+\beta)} - 1}{1 - \frac{(\alpha - r)}{(1+\alpha)} \left(\frac{1}{1-\eta}\right)^{\beta/(1+\beta)}} \right] Y_1, \quad (30)$$

$$K_2 = Y_1 + D_2 - C_1$$

$$= \frac{\beta}{1+\beta}(Y_1 + D_2) + \frac{(1+r)D_2}{(1+\beta)(1+\alpha)}$$

$$K_2 = \frac{\beta}{1+\beta}(Y_1 + D_2).$$

$$U_1 = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1+r)D_2] \tag{31}$$

$$D_2 \leq \bar{D}. \tag{32}$$

$$\mathcal{L} = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1+r)D_2] - \lambda(D_2 - \bar{D})$$

$$u'(C_1) = (1+r)\beta u'(C_2) + \lambda,$$

$$u'(C_1) = [1 + F'(K_2)]\beta u'(C_2),$$

$$\lambda(\bar{D} - D_2) = 0.$$

$$(1 + r)D_2 \leq \eta[F(K_2) + K_2]. \tag{33}$$

$$\mathcal{L} = u(Y_1 + D_2 - K_2) + \beta u[F(K_2) + K_2 - (1 + r)D_2] - \lambda \{(1 + r)D_2 - \eta[F(K_2) + K_2]\}.$$

$$u'(C_1) = (1 + r)[\beta u'(C_2) + \lambda], \tag{34}$$

$$u'(C_1) = [\beta u'(C_2) + \lambda\eta][1 + F'(K_2)], \tag{35}$$

$$\lambda \{\eta[F(K_2) + K_2] - (1 + r)D_2\} = 0,$$

$$\frac{u'(C_1)}{\beta u'(C_2)} = \frac{C_2}{\beta C_1} = \frac{(1 + r)(1 + \alpha)(1 - \eta)}{1 + r - \eta(1 + \alpha)} > 1 + \alpha.$$

$$\eta(1+\alpha)K^{\mathrm{P}} = (1+r)D^{\mathrm{P}}.$$

$$U_1 = C_1 + \mathrm{E}(C_2)$$

$$C_1 = Y_1 - K_2, \quad C_2 = AF(K_2) - \min[\eta AF(K_2), D],$$

$$U_1 = U(K_2) = Y_1 - K_2$$
$$+ \mathrm{E}\left\{AF(K_2) - \min[\eta AF(K_2), D]\right\}$$

$$\max_{K_2} U(K_2) = Y_1 - K_2 + F(K_2) - V(D, K_2) \tag{36}$$

$$V(D, K_2) = \eta F(K_2) \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A + D \int_{\frac{D}{\eta F(K_2)}}^{\overline{A}} \pi(A)\mathrm{d}A \tag{37}$$

$$F'(K_2) \left[1 - \eta \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A \right] = 1. \tag{38}$$

$$
\frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D} = \int_{\frac{D}{\eta F(K_2)}}^{\overline{A}} \pi(A)\mathrm{d}A
$$

$$
+ \left[\eta F'(K_2) \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A \right] K'(D) \tag{39}
$$

$$
p = \frac{V(D, K_2)}{D}.
$$

$$
U_1 = Y_1 - pQ - K_2 + F(K_2) - V(D - Q, K_2)
$$

$$
= Y_1 - \frac{V[D - Q, K(D - Q)]}{D - Q} Q - K(D - Q)
$$

$$
+ F[K(D - Q)] - V[D - Q, K(D - Q)]
$$

$$\left.\frac{\mathrm{d}U_1}{\mathrm{d}Q}\right|_{Q=0} = -\left\{F'[K(D)] - 1\right\} K'(D)$$

$$- \left\{\frac{V[D, K(D)]}{D} - \frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D}\right\} \tag{40}$$

$$- \left\{\frac{V[D, K(D)]}{D} - \frac{\mathrm{d}V[D, K(D)]}{\mathrm{d}D}\right\}$$

$$\left.\frac{\mathrm{d}U_1}{\mathrm{d}Q}\right|_{Q=0} = -\frac{\eta F(K_2)}{D} \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)\mathrm{d}A < 0$$

$$Y = \frac{\underline{Y} + \overline{Y}}{2}.$$

$$\mathrm{E}U_1 = \mathrm{E}\{\log(C_1) + \log(C_2)\},$$

$$C_1^H = C_2^H = \frac{1}{2}(\overline{Y} + Y)$$

$$C_1^L = C_2^L = \frac{1}{2}(\underline{Y} + Y)$$

$$\frac{1}{2}(\overline{Y} - Y) = \frac{1}{2}(Y - \underline{Y}),$$

$$\mathrm{E}U_1 = \frac{1}{2}[\log(\overline{Y} - P_1) + \log(Y - P_2)]$$
$$+ \frac{1}{2}[\log(\underline{Y} + P_1) + \log(Y + P_2)]$$

$$\log(\overline{Y} - P_1) + \log(Y - P_2)$$
$$\geq \log(\overline{Y} + P_1) + \log(Y + P_2) \tag{41}$$

$$\log(\underline{Y} + P_1) + \log(Y + P_2) \geq \log(\underline{Y} - P_1) + \log(Y - P_2) \tag{42}$$

$$P_1 = \frac{\overline{Y}}{\overline{Y} + Y}(\overline{Y} - Y), \qquad P_2 = \frac{-Y}{\overline{Y} + Y}(\overline{Y} - Y), \tag{43}$$

$$\left(\frac{\overline{Y}}{\overline{Y} + Y}\right) 2Y, \qquad \left(\frac{Y}{\overline{Y} + Y}\right) 2Y.$$

$$U_1 = U(C_1, C_2) = C_2.$$

$$Y_2 = \begin{cases} Z & \text{with probability } \pi(I) \\ 0 & \text{with probability } 1 - \pi(I). \end{cases}$$

$$-I + \frac{\pi(I)Z}{1+r},$$

$$\pi'(\bar{I})Z = 1 + r. \tag{44}$$

$$\bar{I} > Y_1,$$

$$I + L = Y_1 + D, \tag{45}$$

$$L \geq 0, \qquad D \geq 0.$$

$$\pi(\bar{I})P(Z) = (1 + r)(\bar{I} - Y_1).$$

$$\begin{aligned} \mathrm{E}C_2 &= \pi(I)[Z - P(Z)] \\ &\quad - [1 - \pi(I)]P(0) + (1 + r)L \\ &= \pi(I)[Z - P(Z)] - [1 - \pi(I)]P(0) \\ &\qquad + (1 + r)(Y_1 + D - I), \end{aligned} \tag{46}$$

$$\pi'(I)\,\{Z - [P(Z) - P(0)]\} = 1 + r. \tag{47}$$

$$\pi(I)P(Z) + [1 - \pi(I)]P(0) = (1 + r)D, \tag{48}$$

$$P(Z) = Z - \frac{1 + r}{\pi'(I)},$$

$$P(Z) = \frac{(1 + r)(I - Y_1)}{\pi(I)}. \tag{49}$$

$$\pi'(I)Z > 1 + r.$$

$$\pi'(I)Z = 1 + r, \qquad \pi'(I^*)Z = 1 + r,$$

$$\pi'(\bar{I})Z = \pi'(\bar{I}^*)Z = \pi'\left[\frac{y_1 + y_1^*}{2(1-s)}\right]Z.$$

$$P(Z) = Z - \frac{1+r}{\pi'(I)}, \quad P(Z)^* = Z - \frac{1+r}{\pi'(I^*)} \tag{50}$$

$$P(Z) = \frac{(1+r)(I - y_1)}{\pi(I)},$$

$$P(Z)^* = \frac{(1+r)(I^* - y_1^*)}{\pi(I^*)} \tag{51}$$

$$1 + r = \frac{\pi'(I)Z}{1 + \frac{\pi'(I)(I - y_1)}{\pi(I)}} \equiv \rho(I, y_1). \tag{52}$$

$$\rho(I, y_1) = \rho(I^*, y_1^*), \tag{53}$$

$$\frac{y_1 + y_1^*}{1 - s} = I + I^*.$$

$$U_1 = U(C_1, C_2) = u(C_2).$$

$$C_2 = \begin{cases} Z - P(Z) + (1+r)(Y_1 - I) & \text{with probability } \pi(I) \\ -P(0) + (1+r)(Y_1 - I) & \text{with probability } 1 - \pi(I). \end{cases}$$

$$P(Z) = [1 - \pi(\bar{I})]Z, \qquad P(0) = -\pi(\bar{I})Z,$$

$$\pi(\bar{I})P(Z) + [1 - \pi(\bar{I})]P(0) = 0.$$

$$\left(\frac{I}{K}\right)_t^i = a_0 + a_1 q_t^i + a_2 \left(\frac{cashflow}{K}\right)_t^i + \epsilon_t^i,$$

$$U_t = \sum_{s=0}^{\infty} \frac{hC_{t+hs}}{(1+\delta h)^s}, \tag{54}$$

$$S_{t+h} = (1 - \theta h)S_t,$$

$$D = \frac{PYh}{rh} = \frac{PY}{r}. \tag{55}$$

$$\frac{\delta + \theta}{(\delta + \theta) + (r + \theta)} = \frac{\delta + \theta}{\delta + r + 2\theta} \tag{56}$$

$$\frac{r + \theta}{(\delta + \theta) + (r + \theta)} = \frac{r + \theta}{\delta + r + 2\theta}. \tag{57}$$

$$D = \left[\frac{\delta + \theta}{(\delta + \theta) + (r + \theta)}\right] \frac{PY}{r},$$

$$U_1 = u(C_1) + \beta \mathrm{E}\{u(C_2)\}, \qquad \beta < 1,$$

$$Y_2 = \bar{Y} + \epsilon.$$

$$C_2(\epsilon) = \bar{Y} + \epsilon - P(\epsilon) + (1 + r)(\bar{Y} - C_1),$$

$$P(\epsilon) \leq \eta(\bar{Y} + \epsilon) + (1 + r)(\bar{Y} - C_1) \tag{58}$$

$$\mathcal{L} = u(C_1) + \sum_{i=1}^{N} \pi(\epsilon_i)\beta u[\bar{Y} + \epsilon_i$$

$$- P(\epsilon_i) + (1+r)(\bar{Y} - C_1)]$$

$$- \sum_{i=1}^{N} \lambda(\epsilon_i)[P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i)$$

$$- (1+r)(\bar{Y} - C_1)] + \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i).$$

$$u'(C_1) = \beta(1+r)\sum_{i=1}^{N}\pi(\epsilon_i)u'[C_2(\epsilon_i)] + (1+r)\sum_{i=1}^{N}\lambda(\epsilon_i) \tag{59}$$

$$\pi(\epsilon)\beta u'[C_2(\epsilon)] + \lambda(\epsilon) = \mu\pi(\epsilon), \tag{60}$$

$$\lambda(\epsilon)[\eta(\bar{Y}+\epsilon) + (1+r)(\bar{Y} - C_1) - P(\epsilon)] = 0, \tag{61}$$

$$u'(C_1) = \beta(1+r)\sum_{i=1}^{N}\pi(\epsilon_i)u'[C_2(\epsilon_i)]$$

$$+ (1+r)\sum_{i=1}^{N}\lambda(\epsilon_i) = (1+r)\mu$$

$$u'(C_1) = (1+r)\beta u'[C_2(\epsilon)] = u'[C_2(\epsilon)] \qquad (62)$$

$$\begin{aligned} P(\epsilon) &= \eta(\bar{Y}+\epsilon) - (1+r)(C_1 - \bar{Y}), \\ C_2(\epsilon) &= (1-\eta)(\bar{Y}+\epsilon) \end{aligned} \qquad (63)$$

$$C_1 = (1 - \eta)(\bar{Y} + e). \tag{64}$$

$$C_2(\epsilon) = \bar{Y} + \epsilon - P(\epsilon) + (1 + r)(\bar{Y} - C_1) = C_1$$

$$P(\epsilon) = \begin{cases} \epsilon + (2 + r)[\eta \bar{Y} - (1 - \eta)e], & \epsilon \in [\underline{\epsilon}, e) \\ \eta\epsilon + (2 + r)\left[\eta \bar{Y} - \dfrac{1 + r}{2 + r}(1 - \eta)e\right], & \epsilon \in [e, \bar{\epsilon}]. \end{cases} \tag{65}$$

$$P(\epsilon) = \begin{cases} \epsilon - (2 + r)e, & \epsilon \in [-\bar{\epsilon}, e) \\ -(1 + r)e, & \epsilon \in [e, \bar{\epsilon}]. \end{cases} \tag{66}$$

$$\int_{-\bar{\epsilon}}^{e} [\epsilon - (2+r)e] \frac{\mathrm{d}\epsilon}{2\bar{\epsilon}} - \int_{e}^{\bar{\epsilon}} (1+r)e \frac{\mathrm{d}\epsilon}{2\bar{\epsilon}} = 0,$$

$$e^2 + [2(3+2r)\bar{\epsilon}]e + \bar{\epsilon}^2 = 0.$$

$$e = -\bar{\epsilon}[3 + 2r - \sqrt{(3+2r)^2 - 1}] \in (-\bar{\epsilon}, 0).$$

$$\begin{aligned} C_2(\epsilon) &= \bar{Y} + \epsilon - P(\epsilon) - (1+r)e \\ &= \bar{Y} + \epsilon - [\epsilon - (2+r)e] - (1+r)e \\ &= \bar{Y} + e. \end{aligned}$$

Table 6.1 Output Processes and Cost of Capital-Market Exclusion, 1950–92

Country	g	$\mathrm{Var}(\epsilon)^{1/2}$	**Cost/Y (κ)**	**Cost per Year (τ)**
Argentina	0.015	0.099	0.36	0.020
Brazil	0.040	0.117	0.24	0.028
Colombia	0.023	0.050	0.04	0.005
Lesotho	0.053	0.160	0.53	0.052
Mexico	0.030	0.088	0.13	0.016
Philippines	0.023	0.100	0.24	0.020
Thailand	0.043	0.081	0.08	0.013
Venezuela	0.011	0.118	Undefined	0.028

Source: Penn World Table, version 5.6. The calculations assume $\beta = 0.95$ and $\rho = 4$.

Table 6.2 Bolivia's March 1988 Debt Buyback

	Prebuyback	**Postbuyback**
Face value of debt, D	\$670 million	\$362 million
Price, p (fraction of a dollar)	0.06	0.11
Total market value, $p \times D$	\$40.2 million	\$39.8 million