5 Uncertainty and International Financial Markets

$$U_1 = \pi(1)\{u(C_1) + \beta u[C_2(1)]\} + \pi(2)\{u(C_1) + \beta u[C_2(2)]\}$$

$$U_1 = u(C_1) + \pi(1)\beta u[C_2(1)] + \pi(2)\beta u[C_2(2)]. \quad (1)$$

$$\frac{p(1)}{1+r}B_2(1) + \frac{p(2)}{1+r}B_2(2) = Y_1 - C_1. \quad (2)$$

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$$C_2(s) = Y_2(s) + B_2(s), \qquad s = 1, 2. \tag{3}$$

$$C_1 + \frac{p(1)C_2(1) + p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r} \tag{4}$$

$$C_2 = p(1)Y_2(1) + p(2)Y_2(2)$$

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$$U_1 = u\left[Y_1 - \frac{p(1)}{1+r}B_2(1) - \frac{p(2)}{1+r}B_2(2)\right] + \sum_{s=1}^{2} \pi(s)\beta u[Y_2(s) + B_2(s)]$$

$$\frac{p(s)}{1+r}u'(C_1) = \pi(s)\beta u'[C_2(s)], \quad s = 1, 2. \qquad (5)$$

$$\frac{\pi(s)\beta u'[C_2(s)]}{u'(C_1)} = \frac{p(s)}{(1+r)}, \quad s = 1, 2 \qquad (6)$$

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$$\frac{(1+r)p(1)}{1+r} + \frac{(1+r)p(2)}{1+r} = 1,$$

$$p(1) + p(2) = 1. \tag{7}$$

$$\begin{aligned}[p(1) + p(2)]u'(C_1) \\ = (1+r)\{\pi(1)\beta u'[C_2(1)] \\ + \pi(2)\beta u'[C_2(2)]\}\end{aligned}$$

$$u'(C_1) = (1+r)\beta \mathrm{E}_1\{u'(C_2)\}, \tag{8}$$

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$$\frac{\beta \mathrm{E}_1\{u'(C_2)\}}{u'(C_1)} = \frac{1}{1+r}.$$

$$\frac{\pi(1)u'[C_2(1)]}{\pi(2)u'[C_2(2)]} = \frac{p(1)}{p(2)}. \tag{9}$$

$$\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)} \tag{10}$$

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$$\mathrm{d}\log\left[\frac{p(1)}{p(2)}\right] = \frac{u''[C(1)]}{u'[C(1)]}\mathrm{d}C(1) - \frac{u''[C(2)]}{u'[C(2)]}\mathrm{d}C(2)$$

$$= \frac{C(1)u''[C(1)]}{u'[C(1)]}\mathrm{d}\log C(1) \tag{11}$$

$$- \frac{C(2)u''[C(2)]}{u'[C(2)]}\mathrm{d}\log C(2)$$

$$\rho(C) = -\frac{Cu''(C)}{u'(C)} \tag{12}$$

$$\mathrm{d}\log\left[\frac{C(2)}{C(1)}\right] = \frac{1}{\rho}\mathrm{d}\log\left[\frac{p(1)}{p(2)}\right]$$

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$$u(C) = \begin{cases} \dfrac{C^{1-\rho}}{1-\rho} & (\rho > 0, \rho \neq 1) \\ \log(C) & (\rho = 1), \end{cases} \tag{13}$$

$$U_1 = \log(C_1) + \pi(1)\beta \log[C_2(1)] + \pi(2)\beta \log[C_2(2)] \tag{14}$$

$$W_1 = Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}.$$

$$C_1 = \frac{1}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right] \tag{15}$$

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$$\frac{p(s)}{1+r}C_2(s) = \frac{\pi(s)\beta}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right]$$

$$s = 1, 2 \tag{16}$$

$$CA_1 = Y_1 - C_1 = \frac{\beta}{1+\beta}Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)}{1+r}Y_2(1) + \frac{p(2)}{1+r}Y_2(2)\right] \tag{17}$$

$$\frac{p(s)^{\mathrm{A}}}{1+r^{\mathrm{A}}} = \frac{\pi(s)u'[Y_2(s)]}{u'(Y_1)},$$

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$$C_1 - Y_1 + \sum_{s=1}^{\mathcal{S}} \frac{p(s)}{1+r}[C_2(s) - Y_2(s)] = 0 \quad (18)$$

$$C_1 - Y_1 + \sum_{s=1}^{\mathcal{S}} \frac{p(s)^{\mathrm{A}}}{1+r^{\mathrm{A}}}[C_2(s) - Y_2(s)] \geq 0 \quad (19)$$

$$\sum_{s=1}^{\mathcal{S}} \left[\frac{p(s)^{\mathrm{A}}}{1+r^{\mathrm{A}}} - \frac{p(s)}{1+r}\right][C_2(s) - Y_2(s)]$$

$$= \sum_{s=1}^{\mathcal{S}} \left[\frac{p(s)^{\mathrm{A}}}{1+r^{\mathrm{A}}} - \frac{p(s)}{1+r}\right] B_2(s) \geq 0$$

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$$U_1 = u(C_1) + \beta u\{\Omega[C_2(1), \ldots, C_2(\mathcal{S}); \pi(1), \ldots, \pi(\mathcal{S})]\} \tag{20}$$

$$U_1 = u(C_1) + \beta u(Z_2/P) \tag{21}$$

$$C_1 + \frac{Z_2}{1+r} = Y_1 + \frac{1}{1+r}\sum_{s=1}^{\mathcal{S}} p(s) Y_2(s) \tag{22}$$

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$$\Omega[C_2(1), \ldots, C_2(\mathcal{S}); \pi(1), \ldots, \pi(\mathcal{S})]$$

$$= \left[\sum_{s=1}^{\mathcal{S}} \pi(s) C_2(s)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \qquad (23)$$

$$C_2(s) = \left[\frac{p(s)/\pi(s)}{P} \right]^{-1/\rho} \frac{Z_2}{P}, \qquad (24)$$

$$P = \left[\sum_{s=1}^{\mathcal{S}} \pi(s)^{\frac{1}{\rho}} p(s)^{\frac{\rho-1}{\rho}} \right]^{\rho/(\rho-1)}. \qquad (25)$$

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$$U_1 = \frac{C_1^{1-1/\sigma}}{1-\frac{1}{\sigma}} + \beta \frac{\left\{\left[\sum_{s=1}^{\mathcal{S}} \pi(s) C_2(s)^{1-\rho}\right]^{\frac{1}{1-\rho}}\right\}^{1-1/\sigma}}{1-\frac{1}{\sigma}} \tag{26}$$

$$U_1 = \frac{C_1^{1-\rho}}{1-\rho} + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) \frac{C_2(s)^{1-\rho}}{1-\rho}.$$

$$u'(C_1) = (1+r)\beta \left(\frac{1}{P}\right) u'\left(\frac{Z_2}{P}\right).$$

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$$Z_2 = (1+r)^{\sigma}\beta^{\sigma}\left(\frac{1}{P}\right)^{\sigma-1} C_1, \tag{27}$$

$$C_1 = \frac{W_1}{1 + \left(\frac{1+r}{P}\right)^{\sigma-1}\beta^{\sigma}}.$$

$$C_1 + C_1^* = Y_1 + Y_1^* \tag{28}$$

$$C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s), \qquad s = 1, 2, \ldots, \mathcal{S} \tag{29}$$

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$$Y_2^{\mathrm{W}}(s) = \left[\frac{\pi(s)\beta(1+r)}{p(s)}\right]^{\frac{1}{\rho}} Y_1^{\mathrm{W}}, \qquad s = 1, 2, \ldots, \mathcal{S},$$

$$\frac{p(s)}{1+r} = \pi(s)\beta \left[\frac{Y_2^{\mathrm{W}}(s)}{Y_1^{\mathrm{W}}}\right]^{-\rho}, \qquad s = 1, 2, \ldots, \mathcal{S} \tag{30}$$

$$\frac{p(s)}{p(s')} = \left[\frac{Y_2^{\mathrm{W}}(s)}{Y_2^{\mathrm{W}}(s')}\right]^{-\rho} \times \frac{\pi(s)}{\pi(s')} \tag{31}$$

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$$p(s') = 1 - \sum_{s \neq s'} p(s)$$

$$= 1 - p(s') \sum_{s \neq s'} \left[\frac{Y_2^{\mathrm{W}}(s)}{Y_2^{\mathrm{W}}(s')} \right]^{-\rho} \frac{\pi(s)}{\pi(s')}$$

$$p(s') = \frac{\pi(s')[Y_2^{\mathrm{W}}(s')]^{-\rho}}{\sum_{s=1}^{\mathcal{S}} \pi(s)[Y_2^{\mathrm{W}}(s)]^{-\rho}}. \tag{32}$$

$$1 + r = \frac{(Y_1^{\mathrm{W}})^{-\rho}}{\beta \sum_{s=1}^{\mathcal{S}} \pi(s)[Y_2^{\mathrm{W}}(s)]^{-\rho}}. \tag{33}$$

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$$\frac{\pi(s)\beta u'[C_2(s)]}{u'(C_1)} = \frac{p(s)}{(1+r)} = \frac{\pi(s)\beta u'[C_2^*(s)]}{u'(C_1^*)} \tag{34}$$

$$\frac{\pi(s)u'[C_2(s)]}{\pi(s')u'[C_2(s')]} = \frac{p(s)}{p(s')} = \frac{\pi(s)u'[C_2^*(s)]}{\pi(s')u'[C_2^*(s')]}$$

$$\frac{C_2(s)}{C_2(s')} = \frac{C_2^*(s)}{C_2^*(s')} = \frac{Y_2^{\mathrm{W}}(s)}{Y_2^{\mathrm{W}}(s')} \tag{35}$$

$$\frac{C_2(s)}{C_1} = \frac{C_2^*(s)}{C_1^*} = \frac{Y_2^{\mathrm{W}}(s)}{Y_1^{\mathrm{W}}} \tag{36}$$

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$$\frac{C_2(s)}{Y_2^{\mathrm{W}}(s)} = \frac{C_2(s')}{Y_2^{\mathrm{W}}(s')}, \qquad \frac{C_2^*(s)}{Y_2^{\mathrm{W}}(s)} = \frac{C_2^*(s')}{Y_2^{\mathrm{W}}(s')}$$

$$\frac{C_2(s)}{Y_2^{\mathrm{W}}(s)} = \mu = \frac{C_1}{Y_1^{\mathrm{W}}}, \qquad \frac{C_2^*(s)}{Y_2^{\mathrm{W}}(s)} = 1 - \mu = \frac{C_1^*}{Y_1^{\mathrm{W}}}$$

$$\log\left[\frac{c_2^n(s)}{c_1^n}\right] = \left(\frac{\rho_m}{\rho_n}\right)\log\left[\frac{c_2^m(s)}{c_1^m}\right] + \frac{1}{\rho_n}\log\left(\frac{\beta_n}{\beta_m}\right) \tag{37}$$

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$$u(c^i) = \frac{(a_0 + a_1 c^i)^{1-\rho}}{1-\rho}, \tag{38}$$

$$(a_0 + a_1 c_1^i)^{-\rho} = \frac{\beta(1+r)\pi(s)[a_0 + a_1 c_2^i(s)]^{-\rho}}{p(s)},$$

$$i = 1, 2, \ldots, I$$

$$a_0 + a_1 c_1^i = \left[\frac{\beta(1+r)\pi(s)}{p(s)}\right]^{-1/\rho} [a_0 + a_1 c_2^i(s)],$$

$$i = 1, 2, \ldots, I$$

$$(a_0 + a_1 c_1)^{-\rho} = \frac{\beta(1+r)\pi(s)[a_0 + a_1 c_2(s)]^{-\rho}}{p(s)}. \tag{39}$$

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$$\tilde{c} \equiv \prod_{i=1}^{I} (c^i)^{1/I}.$$

$$c_2^i(s) = \left[\frac{\pi(s)(1+r)\beta}{p(s)} \right]^{1/\rho_i} c_1^i.$$

$$\tilde{c}_2(s) = \prod_{i=1}^{I} \left[\frac{\pi(s)(1+r)\beta}{p(s)} \right]^{1/I\rho_i} \tilde{c}_1,$$

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$$\frac{p(s)}{1+r} = \pi(s)\beta\left[\frac{\tilde{c}_2(s)}{\tilde{c}_1}\right]^{-\widetilde{\rho}},$$

$$\widetilde{\rho} \equiv \frac{1}{\frac{1}{I}\sum_{i=1}^{I}\frac{1}{\rho_i}}.$$

$$Y_{t+1} = \bar{Y} + \epsilon_{t+1},$$

$$Y^*_{t+1} = \bar{Y} + \epsilon^*_{t+1},$$

$$Y_2(s) = A(s)F(K_2),$$

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$$\sum_{s=1}^{\mathcal{S}} \frac{p(s)}{1+r}[A(s)F(K_2) + K_2] - K_2,$$

$$\sum_{s=1}^{\mathcal{S}} \frac{p(s)}{1+r}[A(s)F'(K_2) + 1] = 1. \tag{40}$$

$$\sum_{s=1}^{\mathcal{S}} p(s)A(s)F'(K_2) = \sum_{s=1}^{\mathcal{S}} p(s)A^*(s)F^{*\prime}(K_2^*) = r.$$

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$$u'(C_1) = \sum_{s=1}^{\mathcal{S}} \pi(s)\beta u'[C_2(s)][A(s)F'(K_2) + 1]$$

$$= \sum_{s=1}^{\mathcal{S}} \pi(s)\beta u'[C_2(s)][A^*(s)F^{*\prime}(K_2^*) + 1] \quad (41)$$

$$Y_1 + Y_1^* = C_1 + C_1^* + K_2 + K_2^*,$$

$$Y_2(s) + Y_2^*(s) + K_2 + K_2^* = C_2(s) + C_2^*(s),$$

$$s = 1, 2, \ldots, \mathcal{S}$$

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$$\frac{p(s)}{1+r} = \frac{\pi(s)\beta(Y_1^{\mathrm{W}} - K_2 - K_2^*)}{Y_2^{\mathrm{W}}(s) + K_2 + K_2^*}$$

$$= \frac{\pi(s)\beta(Y_1^{\mathrm{W}} - K_2 - K_2^*)}{A(s)F(K_2) + A^*(s)F^*(K_2^*) + K_2 + K_2^*}$$

$$\sum_{s=1}^{\mathcal{S}} \left[\frac{\pi(s)\beta(Y_1^{\mathrm{W}} - K_2 - K_2^*)}{A(s)K_2 + A^*(s)K_2^* + K_2 + K_2^*} \right] [A(s) + 1] = 1$$

$$\sum_{s=1}^{\mathcal{S}} \left[\frac{\pi(s)\beta(Y_1^{\mathrm{W}} - K_2 - K_2^*)}{A(s)K_2 + A^*(s)K_2^* + K_2 + K_2^*} \right] [A^*(s) + 1] = 1$$

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$$K_2 + K_2^*$$

$$= \sum_{s=1}^{\mathcal{S}} \left[\frac{\pi(s)\beta(Y_1^{\mathrm{W}} - K_2 - K_2^*)}{A(s)K_2 + A^*(s)K_2^* + K_2 + K_2^*} \right] \left\{ [A(s) + 1]K_2 + [A^*(s) + 1]K_2^* \right\}$$

$$= \sum_{s=1}^{\mathcal{S}} \pi(s)\beta[Y_1^{\mathrm{W}} - (K_2 + K_2^*)] = \beta Y_1^{\mathrm{W}} - \beta(K_2 + K_2^*),$$

$$Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m. \qquad (42)$$

$$C_2^n(s) = (1 + r)B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s). \qquad (43)$$

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$$U_1 = u\left[Y_1^n + V_1^n - B_2^n - \sum_{m=1}^{N} x_m^n V_1^m\right]$$

$$+ \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u\left[(1+r)B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s)\right]$$

$$u'(C_1^n) = (1+r)\beta \sum_{s=1}^{\mathcal{S}} \pi(s) u'[C_2^n(s)]$$

$$= (1+r)\beta \mathrm{E}_1\{u'(C_2^n)\}$$

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$$V_1^m u'(C_1^n) = \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u'[C_2^n(s)] Y_2^m(s)$$

$$= \beta \mathrm{E}_1\{u'(C_2^n) Y_2^m\}, \quad m = 1, 2, \ldots, N \tag{44}$$

$$\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{N} (Y_1^m + V_1^m)}, \tag{45}$$

$$C_1^n = \mu^n \sum_{m=1}^{N} Y_1^m = \mu^n Y_1^{\mathrm{w}}, \tag{46}$$

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$$C_2^n(s) = \mu^n \sum_{m=1}^{N} Y_2^m(s) = \mu^n Y_2^{\mathrm{W}}(s),$$

$$s = 1, \ldots, \mathcal{S} \qquad (47)$$

$$x_m^n = \mu^n, \qquad m = 1, \ldots, N, \qquad (48)$$

$$1 + r = \frac{(C_1^n)^{-\rho}}{\beta \sum_{s=1}^{\mathcal{S}} \pi(s) C_2^n(s)^{-\rho}}.$$

$$1 + r = \frac{(Y_1^{\mathrm{W}})^{-\rho}}{\beta \sum_{s=1}^{\mathcal{S}} \pi(s) Y_2^{\mathrm{W}}(s)^{-\rho}}. \qquad (49)$$

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$$V_1^m = \sum_{s=1}^{\mathcal{S}} \pi(s)\beta \left[\frac{Y_2^{\mathrm{w}}(s)}{Y_1^{\mathrm{w}}}\right]^{-\rho} Y_2^m(s)$$

$$= \beta \mathrm{E}_1 \left\{ \left(\frac{Y_2^{\mathrm{w}}}{Y_1^{\mathrm{w}}}\right)^{-\rho} Y_2^m \right\} \quad m = 1, 2, \ldots, N \quad (50)$$

$$V_1^m = \sum_{s=1}^{\mathcal{S}} \frac{p(s) Y_2^m(s)}{1+r}.$$

$$V_1^m = \sum_{s=1}^{\mathcal{S}} \left\{ \frac{\pi(s)\beta u'[C_2^n(s)]}{u'(C_1^n)} \right\} Y_2^m(s).$$

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$$V_1^m = \mathrm{E}_1\left\{\frac{\beta u'(C_2)}{u'(C_1)} Y_2^m\right\}. \tag{51}$$

$$V_1^m = \mathrm{E}_1\left\{\frac{\beta u'(C_2)}{u'(C_1)}\right\} \mathrm{E}_1\{Y_2^m\}$$

$$+ \mathrm{Cov}_1 \left\{\frac{\beta u'(C_2)}{u'(C_1)}, Y_2^m\right\}$$

$$V_1^m = \frac{\mathrm{E}_1\{Y_2^m\}}{1+r} + \mathrm{Cov}_1 \left\{\frac{\beta u'(C_2)}{u'(C_1)}, Y_2^m\right\} \tag{52}$$

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$$r^m = \frac{Y_2^m - V_1^m}{V_1^m}.$$

$$\begin{aligned} \mathrm{E}_1\{r^m\} - r &= -(1+r)\mathrm{Cov}_1\left\{\frac{\beta u'(C_2)}{u'(C_1)}, 1 + r^m\right\} \\ &= -(1+r)\mathrm{Cov}_1\left\{\frac{\beta u'(C_2)}{u'(C_1)}, r^m - r\right\} \end{aligned} \tag{53}$$

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$$V_1^m - K_2^m = \sum_{s=1}^{\mathcal{S}} \frac{p(s)}{1+r}[A^m(s)F^m(K_2^m) + K_2^m] - K_2^m$$

$$= \sum_{s=1}^{\mathcal{S}} \frac{\pi(s)\beta u'[C_2^n(s)]}{u'(C_1^n)}[A^m(s)F^m(K_2^m) + K_2^m] - K_2^m$$

$$= \mathrm{E}_1\left\{\frac{A^m(s)F^m(K_2^m) + K_2^m}{1+r}\right\}$$

$$+ \mathrm{Cov}_1\left\{\frac{\beta u'[C_2^n(s)]}{u'(C_1^n)}, A^m(s)\right\} F^m(K_2^m) - K_2^m,$$

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$$\frac{\mathrm{d}(V_1^m - K_2^m)}{\mathrm{d}K_2^m} =$$

$$\mathrm{E}_1\left\{\frac{A^m(s)F^{m\prime}(K_2^m) + 1}{1 + r}\right\}$$

$$+ \mathrm{Cov}_1\left\{\frac{\beta u'[C_2^n(s)]}{u'(C_1^n)}, A^m(s)\right\} F^{m\prime}(K_2^m) - 1$$

$$\mathrm{E}_1\{r^m\} - r = -(1 + r)\mathrm{Cov}_1\left\{\beta\left(\frac{C_2}{C_1}\right)^{-\rho}, r^m - r\right\} \tag{54}$$

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$$G\left(\frac{C_2}{C_1}, r^m\right) \equiv \beta\left(\frac{C_2}{C_1}\right)^{-\rho}(r^m - \mathrm{E}_1 r^m),$$

$$G\left(\frac{C_2}{C_1}, r^m\right) \approx \beta(r^m - \mathrm{E}_1 r^m)$$

$$- \beta\rho\left(\frac{C_2}{C_1} - 1\right)(r^m - \mathrm{E}_1 r^m)$$

$$\mathrm{E}_1 G\left(\frac{C_2}{C_1}, r^m\right) = \mathrm{Cov}_1\left\{\beta\left(\frac{C_2}{C_1}\right)^{-\rho}, r^m - r\right\}$$

$$\approx -\beta\rho\mathrm{Cov}_1\left\{\frac{C_2}{C_1} - 1, r^m - r\right\}$$

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$$\mathrm{E}_1\left\{r^m\right\} - r = (1+r)\beta\rho\mathrm{Cov}_1\left\{\frac{C_2}{C_1} - 1, r^m - r\right\}$$

$$= (1+r)\beta\rho\kappa\mathrm{Std}_1\left\{\frac{C_2}{C_1} - 1\right\}\mathrm{Std}_1\{r^m - r\}$$

$$\mathrm{E}\{r^m\} - r = 0.0698 - 0.0080 = 0.0618$$

$$u(C_t, D_t) = \frac{(C_t - D_t)^{1-\rho}}{1-\rho},$$

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$$D_t = (1-\delta)D_{t-1} + \delta\zeta C_{t-1}, \quad 0 < \zeta, \delta < 1$$

$$1 + r = \frac{1}{\mathrm{E}_1 \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right\}}$$

$$r \approx \log(1+r) =$$

$$\rho \mathrm{E}_1 \left\{ \log \left(\frac{C_{t+1}}{C_t} \right) \right\} - \frac{\rho^2}{2} \mathrm{Var} \left\{ \log \left(\frac{C_{t+1}}{C_t} \right) \right\} - \log(\beta)$$

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^n) \right\} \tag{55}$$

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$$B_{s+1}^{n} + \sum_{m=1}^{N} x_{m,s+1}^{n} V_{s}^{m} = (1 + r_{s}) B_{s}^{n} + \sum_{m=1}^{N} x_{m,s}^{n} (Y_{s}^{m} + V_{s}^{m}) - C_{s}^{n} \quad (56)$$

$$u'(C_{s}^{n}) V_{s}^{m} = \beta \mathrm{E}_{s} \{ u'(C_{s+1}^{n}) (Y_{s+1}^{m} + V_{s+1}^{m}) \}, \quad (57)$$

$$u'(C_{s}^{n}) = (1 + r_{s+1}) \beta \mathrm{E}_{s} \{ u'(C_{s+1}^{n}) \} \quad (58)$$

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$$V_t^m = \mathrm{E}_t \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)} Y_{t+1}^m \right\}$$

$$+ \mathrm{E}_t \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)} \mathrm{E}_{t+1} \left\{ \frac{\beta u'(C_{t+2})}{u'(C_{t+1})} (Y_{t+2}^m + V_{t+2}^m) \right\} \right\}$$

$$\mathrm{E}_t \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)} \mathrm{E}_{t+1} \left\{ \frac{\beta u'(C_{t+2})}{u'(C_{t+1})} (Y_{t+2}^m + V_{t+2}^m) \right\} \right\}$$

$$= \mathrm{E}_t \left\{ \mathrm{E}_{t+1} \left\{ \frac{\beta^2 u'(C_{t+2})}{u'(C_t)} (Y_{t+2}^m + V_{t+2}^m) \right\} \right\}.$$

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$$V_t^m = \mathrm{E}_t \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)} Y_{t+1}^m \right\}$$

$$+ \mathrm{E}_t \left\{ \frac{\beta^2 u'(C_{t+2})}{u'(C_t)} (Y_{t+2}^m + V_{t+2}^m) \right\}$$

$$\lim_{T \to \infty} \mathrm{E}_t \left\{ \beta^T [u'(C_{t+T})/u'(C_t)] V_{t+T}^m \right\} = 0$$

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$$V_t^m = \mathrm{E}_t \left\{ \sum_{s=t+1}^{\infty} \frac{\beta^{s-t} u'(C_s)}{u'(C_t)} Y_s^m \right\}$$

$$= \sum_{s=t+1}^{\infty} R_{t,s} \mathrm{E}_t \left\{ Y_s^m \right\}$$

$$+ \sum_{s=t+1}^{\infty} \mathrm{Cov}_t \left\{ \frac{\beta^{s-t} u'(C_s)}{u'(C_t)}, Y_s^m \right\} \tag{59}$$

$$R_{t,s} = \mathrm{E}_t \left\{ \frac{\beta^{s-t} u'(C_s)}{u'(C_t)} \right\}, \tag{60}$$

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$$V_t^m = \mathrm{E}_t \left\{ \sum_{s=t+1}^{\infty} \beta^{s-t} \left(\frac{Y_s^{\mathrm{W}}}{Y_t^{\mathrm{W}}} \right)^{-\rho} Y_s^m \right\},$$

$$m = 1, 2, \ldots, N. \tag{61}$$

$$r_t^m = \frac{Y_t^m}{V_{t-1}^m} + \frac{V_t^m - V_{t-1}^m}{V_{t-1}^m},$$

$$U_1 = u(C_{\mathrm{T},1}^n, C_{\mathrm{N},1}^n) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u[C_{\mathrm{T},2}^n(s), C_{\mathrm{N},2}^n(s)] \tag{62}$$

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$$C^n_{\mathrm{T},1} + p^n_{\mathrm{N},1} C^n_{\mathrm{N},1}$$

$$+ \sum_{s=1}^{\mathcal{S}} \frac{p(s) C^n_{\mathrm{T},2}(s) + p^n_{\mathrm{N},2}(s) p(s) C^n_{\mathrm{N},2}(s)}{1+r}$$

$$= Y^n_{\mathrm{T},1} + p^n_{\mathrm{N},1} Y^n_{\mathrm{N},1}$$

$$+ \sum_{s=1}^{\mathcal{S}} \frac{p(s) Y^n_{\mathrm{T},2}(s) + p^n_{\mathrm{N},2}(s) p(s) Y^n_{\mathrm{N},2}(s)}{1+r}$$

(63)

$$\frac{\partial u(C^n_{\mathrm{T}}, C^n_{\mathrm{N}})/\partial C^n_{\mathrm{N}}}{\partial u(C^n_{\mathrm{T}}, C^n_{\mathrm{N}})/\partial C^n_{\mathrm{T}}} = p^n_{\mathrm{N}}, \tag{64}$$

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$$\frac{p(s)}{1+r} \cdot \frac{\partial u(C^n_{\mathrm{T},1}, C^n_{\mathrm{N},1})}{\partial C^n_{\mathrm{T}}} = \pi(s)\beta \frac{\partial u[C^n_{\mathrm{T},2}(s), C^n_{\mathrm{N},2}(s)]}{\partial C^n_{\mathrm{T}}},$$

$$\frac{1}{p^n_{\mathrm{N},1}} \cdot \frac{p^n_{\mathrm{N},2}(s)\, p(s)}{1+r} \cdot \frac{\partial u(C^n_{\mathrm{T},1}, C^n_{\mathrm{N},1})}{\partial C^n_{\mathrm{N}}} = \pi(s)\beta \frac{\partial u[C^n_{\mathrm{T},2}(s), C^n_{\mathrm{N},2}(s)]}{\partial C^n_{\mathrm{N}}} \tag{65}$$

$$C^n_{\mathrm{N}} = Y^n_{\mathrm{N}}. \tag{66}$$

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$$\frac{\pi(s)\beta\partial u[C^m_{\mathrm{T},2}(s), Y^m_{\mathrm{N},2}(s)]/\partial C^m_{\mathrm{T}}}{\partial u(C^m_{\mathrm{T},1}, Y^m_{\mathrm{N},1})/\partial C^m_{\mathrm{T}}}$$

$$= \frac{\pi(s)\beta\partial u[C^n_{\mathrm{T},2}(s), Y^n_{\mathrm{N},2}(s)]/\partial C^n_{\mathrm{T}}}{\partial u(C^n_{\mathrm{T},1}, Y^n_{\mathrm{N},1})/\partial C^n_{\mathrm{T}}} \tag{67}$$

$$u(C_{\mathrm{T}}, C_{\mathrm{N}}) = \frac{C_{\mathrm{T}}^{1-\rho}}{1-\rho} + v(C_{\mathrm{N}}),$$

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$$V^m_{\mathrm{T},1} = \sum_{s=1}^{\mathcal{S}} \frac{p(s)Y^m_{\mathrm{T},2}(s)}{1+r}$$

$$= \sum_{s=1}^{\mathcal{S}} \frac{\pi(s)\beta\partial u[C^n_{\mathrm{T},2}(s), Y^n_{\mathrm{N},2}(s)]/\partial C^n_{\mathrm{T}}}{\partial u(C^n_{\mathrm{T},1}, Y^n_{\mathrm{N},1})/\partial C^n_{\mathrm{T}}} Y^m_{\mathrm{T},2}(s) \tag{68}$$

$$V^m_{\mathrm{N},1} = \sum_{s=1}^{\mathcal{S}} \frac{p(s)p^m_{\mathrm{N},2}(s)Y^m_{\mathrm{N},2}(s)}{1+r}$$

$$= \sum_{s=1}^{\mathcal{S}} \frac{\pi(s)\beta\partial u[C^n_{\mathrm{T},2}(s), Y^n_{\mathrm{N},2}(s)]/\partial C^n_{\mathrm{T}}}{\partial u(C^n_{\mathrm{T},1}, Y^n_{\mathrm{N},1})/\partial C^n_{\mathrm{T}}} p^m_{\mathrm{N},2}(s)Y^m_{\mathrm{N},2}(s) \tag{69}$$

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$$u(C_1^n) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u[C_2^n(s); \varepsilon^n(s)], \tag{70}$$

$$\frac{\pi(s)\beta u'[C_2^m(s); \varepsilon^m(s)]}{u'(C_1^m)} = \frac{\pi(s)\beta u'[C_2^n(s); \varepsilon^n(s)]}{u'(C_1^n)} \tag{71}$$

$$u(C_{\mathrm{T}}, C_{\mathrm{N}}) = \frac{\left\{\left[\gamma^{\frac{1}{\theta}} C_{\mathrm{T}}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{\mathrm{N}}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right\}^{1-\rho}}{1-\rho} \tag{72}$$

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$$\frac{\partial u(C^n_{\mathrm{T},t}, Y^n_{\mathrm{N},t})/\partial C^n_{\mathrm{T}}}{\partial u(C^n_{\mathrm{T},t-1}, Y^n_{\mathrm{N},t-1})/\partial C^n_{\mathrm{T}}} = \lambda_t.$$

$$\widehat{C}^n_{\mathrm{T}} = \frac{-\theta}{1-[\phi(1-\theta\rho)]}\widehat{\lambda} + \frac{(1-\phi)(1-\theta\rho)}{1-[\phi(1-\theta\rho)]}\widehat{Y}^n_{\mathrm{N}} \tag{73}$$

$$\phi \equiv \frac{\gamma^{\frac{1}{\theta}}(C^n_{\mathrm{T}})^{\frac{\theta-1}{\theta}}}{\gamma^{\frac{1}{\theta}}(C^n_{\mathrm{T}})^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}(C^n_{\mathrm{N}})^{\frac{\theta-1}{\theta}}} < 1.$$

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$$\Delta \log(C^n_{\mathrm{T},t} - Y^n_{\mathrm{D},t})$$

$$= \upsilon_t + \psi_1 \Delta \log Y^n_{\mathrm{N},t} + \psi_2 \Delta \log Y^n_{\mathrm{D},t}$$

$$+ \psi_3 \Delta \log(Y^n_{\mathrm{T},t} - Y^n_{\mathrm{D},t}) + \epsilon^n_t$$

$$U_1 = \frac{C_{\mathrm{T},1}^{1-\rho}}{1-\rho} + v(C_{\mathrm{N},1})$$

$$+ \beta \sum_{s=1}^{\mathcal{S}} \pi(\mathrm{s}) \left\{ \frac{C_{\mathrm{T},2}(\mathrm{s})^{1-\rho}}{1-\rho} + v[C_{\mathrm{N},2}(\mathrm{s})] \right\}$$

$$\frac{C^m_{\mathrm{T},2}(\mathrm{s})}{C^m_{\mathrm{T},1}} = \frac{C^n_{\mathrm{T},2}(\mathrm{s})}{C^n_{\mathrm{T},1}} = \frac{Y^{\mathrm{W}}_{\mathrm{T},2}(\mathrm{s})}{Y^{\mathrm{W}}_{\mathrm{T},1}}, \tag{74}$$

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$$x^n_{\mathrm{N},m} = \begin{cases} 1 & (m = n) \\ 0 & (m \neq n). \end{cases}$$

$$V^m_{\mathrm{N},1} = \sum_{\mathrm{s}=1}^{\mathcal{S}} \pi(s)\beta \left[\frac{Y^{\mathrm{W}}_{\mathrm{T},2}(s)}{Y^{\mathrm{W}}_{\mathrm{T},1}}\right]^{-\rho} p^m_{\mathrm{N},2}(\mathrm{s})Y^m_{\mathrm{N},2}(\mathrm{s})$$

$$u(C_{\mathrm{T}}, C_{\mathrm{N}}) = \gamma \log C_{\mathrm{T}} + (1-\gamma) \log C_{\mathrm{N}},$$

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$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\},$$

$$\mathrm{E}_t\{C_s^{1-\rho}\}$$

$$= (1+g)^{(1-\rho)(s-t)} \bar{C}^{1-\rho} \exp\left[-\frac{1}{2}(1-\rho)\rho \mathrm{Var}(\epsilon) \right]$$

$$U_t = \frac{\bar{C}^{1-\rho}}{1-\rho} \left[\frac{1}{1-\beta(1+g)^{1-\rho}} \right] \exp\left[-\frac{1}{2}(1-\rho)\rho \mathrm{Var}(\epsilon) \right]$$

$$\bar{U}_t = \frac{\bar{C}^{1-\rho}}{1-\rho} \left[\frac{1}{1-\beta(1+g)^{1-\rho}} \right]$$

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$$\frac{[(1+\tau)\bar{C}]^{1-\rho}}{1-\rho}\exp\left[-\frac{1}{2}(1-\rho)\rho\ \mathrm{Var}(\epsilon)\right]=\frac{\bar{C}^{1-\rho}}{1-\rho},$$

$$\tau=\left\{\exp\left[\frac{1}{2}(1-\rho)\rho\ \mathrm{Var}(\epsilon)\right]\right\}^{1/(1-\rho)}-1.$$

$$\tau\approx\frac{1}{2}\rho\ \mathrm{Var}(\epsilon). \tag{75}$$

$$U_t=\log c_t^{\mathrm{Y}}+\beta \mathrm{E}_t \log c_{t+1}^{\mathrm{O}}.$$

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$$U_t = \log c_t^{\mathrm{Y}} + \beta \sum_{s=1}^{\mathcal{S}} \pi_t(s) \log c_{t+1}^{\mathrm{O}}(s)$$

$$c_t^{\mathrm{Y}} + \frac{1}{1+r_{t+1}} \sum_{s=1}^{\mathcal{S}} p_t(s) c_{t+1}^{\mathrm{O}}(s)$$

$$= y_t + \frac{1}{1+r_{t+1}} \sum_{s=1}^{\mathcal{S}} p_t(s) y_{t+1}(s)$$

$$c_t^{\mathrm{Y}} = \mu_t(y_t + y_t^*), \quad c_t^{\mathrm{Y}*} = (1-\mu_t)(y_t + y_t^*)$$

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$$c^{\text{O}}_{t+1}(s) = \mu_t[y_{t+1}(s) + y^*_{t+1}(s)],$$

$$c^{\text{O}*}_{t+1}(s) = (1 - \mu_t)[y_{t+1}(s) + y^*_{t+1}(s)],$$

$$\mu_t = \frac{1}{1+\beta}\left[\frac{y_t}{y_t + y^*_t} + \beta \sum_{s=1}^{\mathcal{S}} \pi_t(s) \frac{y_{t+1}(s)}{y_{t+1}(s) + y^*_{t+1}(s)}\right]$$

$$= \frac{1}{1+\beta}\left[\frac{y_t}{y_t + y^*_t} + \beta \mathrm{E}_t\left\{\frac{y_{t+1}}{y_{t+1} + y^*_{t+1}}\right\}\right]$$

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$$1+r_{t+1}=\frac{\dfrac{1}{y_t+y_t^*}}{\beta\mathrm{E}_t\left\{\dfrac{1}{y_{t+1}+y_{t+1}^*}\right\}}$$

$$p_t(s)=\frac{\dfrac{\pi_t(s)}{y_{t+1}(s)+y_{t+1}^*(s)}}{\mathrm{E}_t\left\{\dfrac{1}{y_{t+1}+y_{t+1}^*}\right\}}.$$

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$$c_t = \frac{1}{2}(c_t^{\mathrm{Y}} + c_t^{\mathrm{O}}) = \frac{1}{2}(\mu_t + \mu_{t-1})(y_t + y_t^*)$$

$$= \frac{y_t + y_t^*}{2(1+\beta)} \left[\frac{y_t}{y_t + y_t^*} + \beta \mathrm{E}_t \left\{ \frac{y_{t+1}}{y_{t+1} + y_{t+1}^*} \right\} + \frac{y_{t-1}}{y_{t-1} + y_{t-1}^*} + \beta \mathrm{E}_{t-1} \left\{ \frac{y_t}{y_t + y_t^*} \right\} \right] \quad (76)$$

$$r^m(s) = \frac{Y_2^m(s) - V_1^m}{V_1^m}.$$

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$$\mathbf{R} \equiv \begin{bmatrix} 1+r & 1+r^1(1) & \dots & 1+r^N(1) \\ 1+r & 1+r^1(2) & \dots & 1+r^N(2) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1+r & 1+r^1(\mathcal{S}) & \dots & 1+r^N(\mathcal{S}) \end{bmatrix}$$

$$\text{Rank}(\mathbf{R}) = \mathcal{S} \tag{77}$$

$$\mathbf{a}_s = [\, a_{0s} \quad a_{1s} \quad \dots \quad a_{Ns} \,]^{\mathcal{T}}$$

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$$CA_1 = Y_1 - C_1 = \frac{\beta}{1+\beta} Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right]$$

$$\frac{p(s)}{(1+r)C_1} = \frac{\pi(s)\beta}{C_2(s)}.$$

$$p(1)C_2(1) + p(2)C_2(2) = \beta(1+r)C_1 = C_1,$$

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$$1 + r^{\mathrm{CA}} = \frac{u'(Y_1)}{\beta\left(\sum_{s=1}^{2}\pi(s)u'\left\{\pi(s)[p(1)Y_2(1) + p(2)Y_2(2)]/p(s)\right\}\right)}$$

$$= \frac{1}{\beta Y_1}[p(1)Y_2(1) + p(2)Y_2(2)], \tag{78}$$

$$1 + r^{\mathrm{A}} = \frac{u'(Y_1)}{\beta\sum_{s=1}^{2}\pi(s)u'[Y_2(s)]}$$

$$= \frac{1}{\beta Y_1}\left[\frac{\pi(1)}{Y_2(1)} + \frac{\pi(2)}{Y_2(2)}\right]^{-1} \tag{79}$$

$$\frac{p(s)^{\mathrm{A}}}{1 + r^{\mathrm{A}}} = \frac{\pi(s)\beta Y_1}{Y_2(s)}, \qquad s = 1, 2. \tag{80}$$

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$$CA_1 = \frac{1}{1+\beta}\left[\pi(1)\beta Y_1 - \frac{p(1)Y_2(1)}{1+r} + \pi(2)\beta Y_1 - \frac{p(2)Y_2(2)}{1+r}\right]$$

$$CA_1 = \frac{Y_2(1)}{1+\beta}\left[\frac{p(1)^{\mathrm{A}}}{1+r^{\mathrm{A}}} - \frac{p(1)}{1+r}\right] + \frac{Y_2(2)}{1+\beta}\left[\frac{p(2)^{\mathrm{A}}}{1+r^{\mathrm{A}}} - \frac{p(2)}{1+r}\right] \tag{81}$$

$$\frac{p(1)^{\mathrm{A}}}{p(2)^{\mathrm{A}}} = \frac{\pi(1)/Y_2(1)}{\pi(2)/Y_2(2)}.$$

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$$
\begin{aligned}
B_2(1) &= C_2(1) - Y_2(1) \\
&= \frac{\pi(1)[p(1)Y_2(1) + p(2)Y_2(2)]}{p(1)} - Y_2(1) \\
&= \frac{p(2)\pi(1)}{p(1)} Y_2(2) - \pi(2)Y_2(1) \\
&= \frac{\pi(2)p(2)Y_2(1)}{p(1)} \left[\frac{\pi(1)/Y_2(1)}{\pi(2)/Y_2(2)} - \frac{p(1)}{p(2)} \right] \\
&= \frac{p(2)}{p(1)} \pi(2)Y_2(1) \left[\frac{p(1)^{\mathrm{A}}}{p(2)^{\mathrm{A}}} - \frac{p(1)}{p(2)} \right].
\end{aligned}
$$

$$
B_2(2) = -\pi(2)Y_2(1) \left[\frac{p(1)^{\mathrm{A}}}{p(2)^{\mathrm{A}}} - \frac{p(1)}{p(2)} \right].
$$

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$$U_1 = u(C_1^n) + \sum_{t=2}^{\infty} \beta^{t-1} \left\{ \sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_1)} \pi(\mathbf{h}_t \mid \mathbf{h}_1) u[C^n(\mathbf{h}_t)] \right\} \tag{82}$$

$$C_1^n + \sum_{t=2}^{\infty} R_{1,t} \left[\sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_1)} p(\mathbf{h}_t \mid \mathbf{h}_1) C^n(\mathbf{h}_t) \right] = Y_1^n + \sum_{t=2}^{\infty} R_{1,t} \left[\sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_1)} p(\mathbf{h}_t \mid \mathbf{h}_1) Y^n(\mathbf{h}_t) \right] \tag{83}$$

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$$
\begin{aligned}
&R_{1,t}\, p(\mathbf{h}_t \mid \mathbf{h}_1) u'(C_1^n) \\
&\quad = \pi(\mathbf{h}_t \mid \mathbf{h}_1)\beta^{t-1} u'[C^n(\mathbf{h}_t)]
\end{aligned}
\tag{84}
$$

$$
\frac{\pi(\mathbf{h}_t^1 \mid \mathbf{h}_1) u'[C^n(\mathbf{h}_t^1)]}{\pi(\mathbf{h}_t^2 \mid \mathbf{h}_1) u'[C^n(\mathbf{h}_t^2)]} = \frac{p(\mathbf{h}_t^1 \mid \mathbf{h}_1)}{p(\mathbf{h}_t^2 \mid \mathbf{h}_1)},
$$

$$
\begin{aligned}
u'(C_1^n) &= \frac{\beta^{t-1}}{R_{1,t}} \mathrm{E}_1\{u'(C_t^n)\} \\
&= \frac{\beta^{t-1}}{R_{1,t}} \mathrm{E}\{u'[C^n(\mathbf{h}_t)] \mid \mathbf{h}_1\}
\end{aligned}
\tag{85}
$$

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$$\sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_1)} p(\mathbf{h}_t \mid \mathbf{h}_1) = 1$$

$$C^n(\mathbf{h}_t) = \mu^n Y^{\mathrm{W}}(\mathbf{h}_t),$$

$$R_{1,t} = \frac{\beta^{t-1} \sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_1)} \pi(\mathbf{h}_t \mid \mathbf{h}_1) Y^{\mathrm{W}}(\mathbf{h}_t)^{-\rho}}{(Y_1^{\mathrm{W}})^{-\rho}}$$

$$\widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_1) u'(C_1) = \pi(\mathbf{h}_t \mid \mathbf{h}_1) \beta^{t-1} u'[C(\mathbf{h}_t)]$$

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$$C_2' + \sum_{t=3}^{\infty} \left[\sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_2)} \widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_2) C(\mathbf{h}_t)' \right]$$

$$= C(\mathbf{h}_2) + \sum_{t=3}^{\infty} \left[\sum_{\mathbf{h}_t \in \mathbf{H}_t(\mathbf{h}_2)} \widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_2) C(\mathbf{h}_t) \right]$$

$$\widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_2) u'[C(\mathbf{h}_2)] = \pi(\mathbf{h}_t \mid \mathbf{h}_2) \beta^{t-2} u'[C(\mathbf{h}_t)] \tag{87}$$

$$\widetilde{p}(\mathbf{h}_2 \mid \mathbf{h}_1) u'(C_1) = \pi(\mathbf{h}_2 \mid \mathbf{h}_1) \beta u'[C(\mathbf{h}_2)].$$

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$$\frac{\widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_1)}{\widetilde{p}(\mathbf{h}_2 \mid \mathbf{h}_1)} u'[C(\mathbf{h}_2)] = \left[\frac{\pi(\mathbf{h}_t \mid \mathbf{h}_1)}{\pi(\mathbf{h}_2 \mid \mathbf{h}_1)}\right] \beta^{t-2} u'[C(\mathbf{h}_t)] \tag{88}$$

$$\frac{\pi(\mathbf{h}_t \mid \mathbf{h}_1)}{\pi(\mathbf{h}_2 \mid \mathbf{h}_1)} = \pi(\mathbf{h}_t \mid \mathbf{h}_2).$$

$$\frac{\widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_1)}{\widetilde{p}(\mathbf{h}_2 \mid \mathbf{h}_1)} = \widetilde{p}(\mathbf{h}_t \mid \mathbf{h}_2).$$

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Table 5.1 Consumption and Output: Correlations between Domestic and World Growth Rates, 1973–92

Country	Corr $(\hat{c}, \hat{c}^{w})$	Corr $(\hat{y}, \hat{y}^{w})$
Canada	0.56	0.70
France	0.45	0.60
Germany	0.63	0.70
Italy	0.27	0.51
Japan	0.38	0.46
United Kingdom	0.63	0.62
United States	0.52	0.68
OECD average	0.43	0.52
Developing country average	−0.10	0.05

Note: The numbers Corr$(\hat{c}, \hat{c}^{w})$ and Corr$(\hat{y}, \hat{y}^{w})$ are the simple correlation coefficients between the annual change in the natural logarithm of a country's real per capita consumption (or output) and the annual change in the natural logarithm of the rest of the world's real per capita consumption (or output), with the "world" defined as the 35 benchmark countries in the Penn World Table (version 5.6). Average correlations are population-weighted averages of individual country correlations. The OECD average excludes Mexico.

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Table 5.2 Share of Domestic Equities in Total Equity Portfolio, End 1989

United States	United Kingdom	Japan
0.96	0.82	0.98

Source: French and Poterba (1991).

Table 5.3 Real U.S. Stock and Government Bond Returns (annual geometrically compounded percent rate of return)

Period	Stocks	Short Bonds	Long Bonds
1802–1992	6.7	2.9	3.4
1871–1992	6.6	1.7	2.6
1802–70	7.0	5.1	4.8
1871–1925	6.6	3.2	3.7
1926–92	6.6	0.5	1.7
1946–92	6.6	0.4	0.4

Source: Siegel (1995).

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Table 5.4 Measures of V^m, the Securitized Value of a Claim to a Country's Entire Future GDP, 1992 (billions of U.S. dollars)

Country	V^m	Std(r^m)	Country	V^m	Std(r^m)
Argentina	2,460	9.86	Nigeria	2,019	10.06
Australia	4,340	3.88	Pakistan	2,894	2.45
Brazil	10,032	8.88	Philippines	1,602	3.68
Canada	7,663	4.22	South Africa	1,722	8.98
France	12,901	5.38	Spain	6,721	6.30
Germany (West)	16,796	4.47	Sweden	1,972	5.70
India	20,378	4.32	Switzerland	1,911	5.30
Italy	11,540	4.68	Thailand	4,007	3.99
Japan	31,762	8.41	Turkey	3,868	3.38
Kenya	418	4.34	United Kingdom	13,495	1.46
Mexico	9,583	5.33	United States	82,075	2.03
Netherlands	3,607	4.68	Venezuela	2,501	6.87

Source: Methodology is based on Shiller (1993, ch. 4). Underlying annual real GDP data are from Penn World Table, version 5.6. Standard deviations are on annual return (income plus appreciation) of a perpetual claim to GDP.

[a] 1990 value based on 1950–90 data.