4 The Real Exchange Rate and the Terms of Trade

$$Y_{\mathrm{T}} = A_{\mathrm{T}} F(K_{\mathrm{T}}, L_{\mathrm{T}}), \qquad Y_{\mathrm{N}} = A_{\mathrm{N}} G(K_{\mathrm{N}}, L_{\mathrm{N}}) \quad (1)$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_{\mathrm{T},s} F(K_{\mathrm{T},s}, L_{\mathrm{T},s}) - w_s L_{\mathrm{T},s} - \Delta K_{\mathrm{T},s+1}\right]$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[p_s A_{\mathrm{N},s} G(K_{\mathrm{N},s}, L_{\mathrm{N},s}) - w_s L_{\mathrm{N},s} - \Delta K_{\mathrm{N},s+1}\right]$$

$$A_{\mathrm{T}} f'(k_{\mathrm{T}}) = r \tag{2}$$

$$A_{\mathrm{T}} \left[f(k_{\mathrm{T}}) - f'(k_{\mathrm{T}}) k_{\mathrm{T}} \right] = w \tag{3}$$

$$p A_{\mathrm{N}} g'(k_{\mathrm{N}}) = r \tag{4}$$

$$p A_{\mathrm{N}} \left[g(k_{\mathrm{N}}) - g'(k_{\mathrm{N}}) k_{\mathrm{N}} \right] = w \tag{5}$$

$$w(r, A_{\mathrm{T}}) = A_{\mathrm{T}} f[k_{\mathrm{T}}(r, A_{\mathrm{T}})] - r k_{\mathrm{T}}(r, A_{\mathrm{T}}). \quad (6)$$

$$p A_{\mathrm{N}} \left[g(k_{\mathrm{N}}) - g'(k_{\mathrm{N}}) k_{\mathrm{N}} \right] = w(r, A_{\mathrm{T}}),$$

$$A_{\mathrm{T}} f(k_{\mathrm{T}}) = r k_{\mathrm{T}} + w, \;\; p A_{\mathrm{N}} g(k_{\mathrm{N}}) = r k_{\mathrm{N}} + w \quad (7)$$

$$\frac{\mathrm{d}A_{\mathrm{T}}}{A_{\mathrm{T}}} + \frac{r k_{\mathrm{T}}}{A_{\mathrm{T}} f(k_{\mathrm{T}})} \frac{\mathrm{d}k_{\mathrm{T}}}{k_{\mathrm{T}}} = \frac{r k_{\mathrm{T}}}{A_{\mathrm{T}} f(k_{\mathrm{T}})} \frac{\mathrm{d}k_{\mathrm{T}}}{k_{\mathrm{T}}} + \frac{w}{A_{\mathrm{T}} f(k_{\mathrm{T}})} \frac{\mathrm{d}w}{w},$$

$$\hat{A}_{\mathrm{T}} = \mu_{\mathrm{LT}}\hat{w}. \tag{8}$$

$$\hat{p} + \hat{A}_{\mathrm{N}} = \mu_{\mathrm{LN}}\hat{w}.$$

$$\hat{p} = \frac{\mu_{\mathrm{LN}}}{\mu_{\mathrm{LT}}}\hat{A}_{\mathrm{T}} - \hat{A}_{\mathrm{N}} \tag{9}$$

$$\hat{p} = \frac{1}{\mu_{\mathrm{LT}}}\left(\mu_{\mathrm{KN}} - \mu_{\mathrm{KT}}\right)\hat{r} = \frac{1}{\mu_{\mathrm{LT}}}\left(\mu_{\mathrm{LT}} - \mu_{\mathrm{LN}}\right)\hat{r}.$$

$$P = (1)^{\gamma} p^{1-\gamma} = p^{1-\gamma}, \quad P^* = (1)^{\gamma} (p^*)^{1-\gamma} = (p^*)^{1-\gamma}$$

$$\frac{P}{P^*} = \left(\frac{p}{p^*}\right)^{1-\gamma}$$

$$\hat{P} - \hat{P}^* = (1 - \gamma)(\hat{p} - \hat{p}^*)$$

$$= (1 - \gamma) \left[\frac{\mu_{\mathrm{LN}}}{\mu_{\mathrm{LT}}} \left(\hat{A}_{\mathrm{T}} - \hat{A}^*_{\mathrm{T}}\right) - \left(\hat{A}_{\mathrm{N}} - \hat{A}^*_{\mathrm{N}}\right)\right]$$

$$\hat{w}_{\mathrm{L}} = \hat{w}_{\mathrm{S}} = \frac{\hat{A}_{\mathrm{T}}}{1 - \mu_{\mathrm{KT}}},$$

$$\hat{p} = \mu_{\mathrm{LN}}\hat{w}_{\mathrm{L}} + \mu_{\mathrm{SN}}\hat{w}_{\mathrm{S}} - \hat{A}_{\mathrm{N}} = \left(\frac{\mu_{\mathrm{LN}} + \mu_{\mathrm{SN}}}{1 - \mu_{\mathrm{KT}}}\right)\hat{A}_{\mathrm{T}} - \hat{A}_{\mathrm{N}}.$$

$$Q \equiv B + K_{\mathrm{T}} + K_{\mathrm{N}} = B + K.$$

$$\bar{Y}_{\mathrm{T}} = r\bar{K}_{\mathrm{T}} + w\bar{L}_{\mathrm{T}} = \left[rk_{\mathrm{T}}(r) + w(r)\right]\bar{L}_{\mathrm{T}},$$

$$p(r)\bar{Y}_{\mathrm{N}} = r\bar{K}_{\mathrm{N}} + w\bar{L}_{\mathrm{N}} = \left[rk_{\mathrm{N}}(r) + w(r)\right]\left(L - \bar{L}_{\mathrm{T}}\right)$$

$$\bar{Y}_{\mathrm{T}} = -\left[\frac{rk_{\mathrm{T}}(r) + w(r)}{rk_{\mathrm{N}}(r) + w(r)}\right]p(r)\bar{Y}_{\mathrm{N}} + \left[rk_{\mathrm{T}}(r) + w(r)\right]L \qquad (10)$$

$$\bar{C}_{\mathrm{T}} + p(r)\bar{C}_{\mathrm{N}} = w(r)L + r\bar{Q}. \tag{11}$$

$$Y_{\mathrm{N}} = A_{\mathrm{N}}K_{\mathrm{N}}^{\alpha}L_{\mathrm{N}}^{1-\alpha} = A_{\mathrm{N}}k_{\mathrm{N}}^{\alpha}L_{\mathrm{N}}.$$

$$\hat{L}_{\mathrm{N}} = \hat{Y}_{\mathrm{N}} - \alpha\hat{k}_{\mathrm{N}};$$

$$\hat{L}_{\mathrm{N}} = \hat{C}_{\mathrm{N}} - \alpha\hat{k}_{\mathrm{N}}. \tag{12}$$

$$\left[\gamma^{\frac{1}{\theta}} C_{\mathrm{T}}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{\mathrm{N}}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{13}$$

$$\gamma \in (0, 1), \quad \theta > 0,$$

$$Z \equiv C_{\mathrm{T}} + pC_{\mathrm{N}}. \tag{14}$$

$$\frac{\gamma C_{\mathrm{N}}}{(1-\gamma)C_{\mathrm{T}}} = p^{-\theta}, \tag{15}$$

$$C_{\mathrm{T}} = \frac{\gamma Z}{\gamma + (1-\gamma)p^{1-\theta}},$$

$$C_{\mathrm{N}} = \frac{p^{-\theta}(1-\gamma)Z}{\gamma + (1-\gamma)p^{1-\theta}} \tag{16}$$

$$\hat{C}_{\mathrm{N}} = \hat{Z} - \left[\gamma\theta + (1-\gamma)\right]\hat{p}. \tag{17}$$

$$\hat{\bar{Z}} = \frac{wL}{wL + r\bar{Q}}\hat{w} \equiv \psi_{\mathrm{L}}\hat{w},$$

$$\hat{\bar{Z}} = \frac{\psi_{\mathrm{L}}}{\mu_{\mathrm{LT}}}\hat{A}_{\mathrm{T}}.$$

$$\hat{p} = \frac{1-\alpha}{\mu_{\mathrm{LT}}} \hat{A}_{\mathrm{T}}$$

$$\widehat{\bar{C}}_{\mathrm{N}} = \left\{ \psi_{\mathrm{L}} - (1-\alpha) \left[\gamma\theta + (1-\gamma) \right] \right\} \frac{\hat{A}_{\mathrm{T}}}{\mu_{\mathrm{LT}}}.$$

$$(1-\alpha)\hat{k}_{\mathrm{N}} = \hat{p} = \frac{1-\alpha}{\mu_{\mathrm{LT}}} \hat{A}_{\mathrm{T}},$$

$$\widehat{\bar{L}}_{\mathrm{N}} = \left\{ \psi_{\mathrm{L}} - (1-\alpha) \left[\gamma\theta + (1-\gamma) \right] - \alpha \right\} \frac{\hat{A}_{\mathrm{T}}}{\mu_{\mathrm{LT}}} \qquad (18)$$

$$\hat{\bar{L}}_{\mathrm{N}} = (\psi_{\mathrm{L}} - 1)\,\frac{\hat{A}_{\mathrm{T}}}{\mu_{\mathrm{LT}}}.$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\,(C_s)\,, \tag{19}$$

$$C = \Omega\,(C_{\mathrm{T}}, C_{\mathrm{N}})$$

$$= \left[\gamma^{\frac{1}{\theta}} C_{\mathrm{T}}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{\mathrm{N}}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \quad \gamma \in (0, 1), \quad \theta > 0$$

$$\left\{ \gamma^{\frac{1}{\theta}} \left[\frac{\gamma Z}{\gamma + (1-\gamma)p^{1-\theta}} \right]^{\frac{\theta-1}{\theta}} \right.$$

$$\left. + (1-\gamma)^{\frac{1}{\theta}} \left[\frac{p^{-\theta}(1-\gamma)Z}{\gamma + (1-\gamma)p^{1-\theta}} \right]^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

$$\left\{ \gamma^{\frac{1}{\theta}} \left[\frac{\gamma P}{\gamma + (1-\gamma)p^{1-\theta}} \right]^{\frac{\theta-1}{\theta}} \right.$$

$$\left. + (1-\gamma)^{\frac{1}{\theta}} \left[\frac{p^{-\theta}(1-\gamma)P}{\gamma + (1-\gamma)p^{1-\theta}} \right]^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}} = 1$$

$$P = \left[\gamma + (1-\gamma) p^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{20}$$

$$C = \frac{Z}{P}. \tag{21}$$

$$C_{\mathrm{T}} = \gamma \left(\frac{1}{P} \right)^{-\theta} C, \qquad C_{\mathrm{N}} = (1-\gamma) \left(\frac{p}{P} \right)^{-\theta} C \tag{22}$$

$$P = (1)^{\gamma}\ p^{1-\gamma} = p^{1-\gamma}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_{\mathrm{T},s}+p_s C_{\mathrm{N},s}) = (1+r)Q_t$$

$$+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s L_s - G_s) \qquad (23)$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}P_s C_s = (1+r)Q_t$$

$$+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s L_s - G_s) \qquad (24)$$

$$Q_{s+1} - Q_s = rQ_s + w_s L_s - G_s - P_s C_s,$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left[\frac{(1+r)Q_s - Q_{s+1} + w_s L_s - G_s}{P_s} \right]$$

$$\frac{u'(C_s)}{P_s} = (1+r)\beta \frac{u'(C_{s+1})}{P_{s+1}}.$$

$$1 + r^{\mathrm{C}}_{s+1} \equiv \frac{(1+r)P_s}{P_{s+1}}. \tag{25}$$

$$u'(C_s) = \frac{(1+r)P_s}{P_{s+1}}\beta u'(C_{s+1}) = \left(1 + r^{\mathrm{C}}_{s+1}\right)\beta u'(C_{s+1}) \tag{26}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\frac{P_s C_s}{P_t} = \sum_{s=t}^{\infty}\left[\frac{P_{t+1}}{(1+r)P_t}\right]\left[\frac{P_{t+2}}{(1+r)P_{t+1}}\right] \cdots \left[\frac{P_s}{(1+r)P_{s-1}}\right]C_s = \sum_{s=t}^{\infty}\frac{C_s}{\prod_{v=t+1}^{s}(1+r^{\mathrm{C}}_v)}$$

$$\frac{(1+r)Q_t}{P_t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{P_s\,(w_s L_s - G_s)}{P_s P_t}$$

$$= \frac{(1+r)Q_t}{P_t} + \sum_{s=t}^{\infty} \frac{(w_s L_s - G_s)\,/P_s}{\prod_{v=t+1}^{s}(1+r_v^{\mathrm{C}})}.$$

$$R_{t,s}^{\mathrm{C}} = \frac{1}{\prod_{v=t+1}^{s}(1+r_v^{\mathrm{C}})},$$

$$\sum_{s=t}^{\infty} R_{t,s}^{\mathrm{C}} C_s = \frac{(1+r)Q_t}{P_t} + \sum_{s=t}^{\infty} R_{t,s}^{\mathrm{C}} \frac{w_s L_s - G_s}{P_s}. \qquad (27)$$

$$C_t = \frac{\frac{(1+r)Q_t}{P_t} + \sum_{s=t}^{\infty} R_{t,s}^{\mathrm{C}} \frac{w_s L_s - G_s}{P_s}}{\sum_{s=t}^{\infty} \left(R_{t,s}^{\mathrm{C}}\right)^{1-\sigma} \beta^{\sigma(s-t)}}. \tag{28}$$

$$C_t = \frac{(1+r)Q_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (w_s L_s - G_s)}{P_t \sum_{s=t}^{\infty} \left[(1+r)^{s-t} \left(\frac{P_t}{P_s}\right)\right]^{\sigma-1} \beta^{\sigma(s-t)}}. \tag{29}$$

$$K_t = K_{\mathrm{T},t} + K_{\mathrm{N},t}$$

$$= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t+1} \left[r\left(K_{\mathrm{T},s} + K_{\mathrm{N},s}\right) - \Delta K_{\mathrm{T},s+1} - \Delta K_{\mathrm{N},s+1}\right]$$

$$= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t+1} (rK_s - I_s)$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} P_s C_s$$

$$= (1+r)\,(B_t + K_t) + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (w_s L_s - G_s)$$

$$= (1+r) B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (Y_{\mathrm{T},s} + p_s Y_{\mathrm{N},s} - I_s - G_s)$$

(30)

$$C_{\mathrm{N}} + G_{\mathrm{N}} = Y_{\mathrm{N}}. \tag{31}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(C_{\mathrm{T},s}+I_s+G_{\mathrm{T},s}\right)$$

$$=(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}Y_{\mathrm{T},s} \tag{32}$$

$$C_{s+1}=\left[\frac{(1+r)P_s}{P_{s+1}}\right]^{\sigma}\beta^{\sigma}C_s. \tag{33}$$

$$C_{\mathrm{T},s+1}=\left(\frac{P_s}{P_{s+1}}\right)^{\sigma-\theta}(1+r)^{\sigma}\beta^{\sigma}C_{\mathrm{T},s}. \tag{34}$$

$$C_{\mathrm{T},t} = \frac{(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(Y_{\mathrm{T},s} - I_s - G_{\mathrm{T},s}\right)}{\sum_{s=t}^{\infty} \left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t} \left(\frac{P_t}{P_s}\right)^{\sigma-\theta}} \tag{35}$$

$$\begin{aligned} CA_t = B_{t+1} - B_t &= rB_t + Y_{\mathrm{T},t} \\ &\quad + p_t Y_{\mathrm{N},t} - C_{\mathrm{T},t} - p_t C_{\mathrm{N},t} - I_t - G_t \\ &= rB_t + Y_{\mathrm{T},t} - C_{\mathrm{T},t} - I_t - G_{\mathrm{T},t} \end{aligned} \tag{36}$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s, \tag{37}$$

$$C = \exp\left[\int_0^1 \log c(z)\mathrm{d}z\right]. \tag{38}$$

$$P = \exp\left[\int_0^1 \log p(z)\mathrm{d}z\right]. \tag{39}$$

$$\min_{\{C(z)|z\in[0,1]\}} \int_0^1 p(z)c(z)\mathrm{d}z$$

$$C = \exp\left[\int_0^1 \log c(z)\mathrm{d}z\right] = 1$$

$$\int_0^1 \log c(z)\mathrm{d}z = 0,$$

$$\mathcal{L} = \int_0^1 p(z)c(z)\mathrm{d}z - \lambda \int_0^1 \log c(z)\mathrm{d}z,$$

$$p(z)c(z) = \lambda,$$

$$\lambda = \exp\left[\int_0^1 \log p(z)\mathrm{d}z\right].$$

$$\int_0^1 p(z)c(z)\mathrm{d}z = \int_0^1 \lambda dz = \lambda = \exp\left[\int_0^1 \log p(z)\mathrm{d}z\right]$$

$$c(z) = \left[\frac{P}{p(z)}\right] C. \tag{40}$$

$$A(z) \equiv \frac{a^*(z)}{a(z)}. \tag{41}$$

$$A'(z) < 0.$$

$$\frac{w}{w^*} < A(z) = \frac{a^*(z)}{a(z)}$$

$$\frac{w}{w^*} > A(z)$$

$$\frac{w}{w^*} = A(\tilde{z}),$$

$$P(C + C^*) = wL + w^*L^*. \tag{42}$$

$$wL = zP(C + C^*) = z(wL + w^*L^*),$$

$$\frac{w}{w^*} = \frac{z}{1-z}\left(\frac{L^*}{L}\right) \equiv B\left(z; \frac{L^*}{L}\right). \tag{43}$$

$$p(z) = a(z)w \tag{44}$$

$$p(z) = a^*(z)w^*. \tag{45}$$

$$\frac{w'}{p(z)'} = \frac{w'}{a^*(z)w^{*\prime}} > \frac{w}{a^*(z)w^*} = \frac{w}{p(z)},$$

$$\frac{w'}{p(z)'} > \frac{1}{a(z)} = \frac{w}{p(z)}.$$

$$\frac{w^{*\prime}}{p(z)'} = \frac{1}{a^*(z)} < \frac{w^*}{p(z)}$$

$$CA_t = B_{t+1} - B_t = \frac{w_t L}{P_t} + r_t B_t - C_t, \tag{46}$$

$$CA_t^* = B_{t+1}^* - B_t^* = \frac{w_t^* L^*}{P_t} + r_t B_t^* - C_t^* \tag{47}$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \log \left[(1 + r_t) B_t - B_{t+1} + \frac{w_t L}{P_t} \right]$$

$$C_{t+1} = (1 + r_{t+1}) \beta C_t. \tag{48}$$

$$\bar{r} = \frac{1 - \beta}{\beta}.$$

$$\bar{C} = \bar{r}\bar{B} + \frac{\bar{w}L}{\bar{P}}, \qquad \bar{C}^* = -\bar{r}\bar{B} + \frac{\bar{w}^*L^*}{\bar{P}} \tag{49}$$

$$\widehat{\bar{C}} = \frac{\bar{r}\,\mathrm{d}\bar{B}}{\bar{C}_0}, \qquad \widehat{\bar{C}^*} = \frac{-\bar{r}\,\mathrm{d}\bar{B}}{\bar{C}_0^*}. \tag{50}$$

$$\log C_{t+1} = \log(1 + r_{t+1}) + \log\beta + \log C_t,$$

$$\frac{\mathrm{d}C_{t+1}}{C_{t+1}} = \frac{\mathrm{d}r_{t+1}}{1 + r_{t+1}} + \frac{\mathrm{d}C_t}{C_t}.$$

$$\widehat{\bar{C}} = (1 - \beta)\hat{r} + \hat{C}, \tag{51}$$

$$\widehat{\bar{C}^*} = (1 - \beta)\hat{r} + \hat{C}^*. \tag{52}$$

$$L = L^* \quad \text{and} \quad A(1/2) = 1,$$

$$\tilde{z}_0 = 1/2$$

$$\frac{\mathrm{d}\bar{C} + \mathrm{d}\bar{C}^*}{\bar{C} + \bar{C}^*} = \left(\frac{\bar{C}}{\bar{C} + \bar{C}^*}\right)\widehat{\bar{C}} + \left(\frac{\bar{C}^*}{\bar{C} + \bar{C}^*}\right)\widehat{\bar{C}^*} = \frac{\widehat{\bar{C}} + \widehat{\bar{C}^*}}{2} = 0$$

$$\hat{r} = \frac{-1}{(1-\beta)}\left(\frac{\hat{C} + \hat{C}^*}{2}\right),$$

$$\frac{\hat{C} + \hat{C}^*}{2} = \frac{\hat{w} + \hat{w}^*}{2} - \hat{P}. \tag{53}$$

$$P = \exp\left[\int_0^{\tilde{z}} \log wa(z)\mathrm{d}z + \int_{\tilde{z}}^1 \log \frac{w^*a^*(z)}{\nu}\mathrm{d}z\right]$$

$$\hat{P} = \frac{\hat{w} + \hat{w}^*}{2} - \frac{\hat{\nu}}{2}. \tag{54}$$

$$\frac{\hat{C} + \hat{C}^*}{2} = \frac{\hat{\nu}}{2},$$

$$\hat{r} = \frac{-\hat{\nu}}{2(1-\beta)}. \tag{55}$$

$$\frac{\mathrm{d}\bar{B}}{\bar{C}_0} = \hat{w} - \hat{P} - \hat{C} = -\frac{\mathrm{d}\bar{B}^*}{\bar{C}_0^*}.$$

$$\frac{\mathrm{d}\bar{B}}{\bar{C}_0} = \frac{\hat{w} - \hat{w}^* + \hat{\nu}}{2} - \hat{C}. \tag{56}$$

$$\hat{w} - \hat{w}^* = -\hat{\nu} + A'(1/2)\mathrm{d}\tilde{z}$$

$$\hat{w} - \hat{w}^* = 4\mathrm{d}\tilde{z},$$

$$\hat{w} - \hat{w}^* = \frac{-\hat{\nu}}{1 - \frac{1}{4}A'\left(\frac{1}{2}\right)}. \tag{57}$$

$$\frac{\mathrm{d}\bar{B}}{\bar{C}_0} = \frac{-A'\left(\frac{1}{2}\right)\hat{\nu}}{8 - 2A'\left(\frac{1}{2}\right)} - \widehat{\bar{C}} + (1-\beta)\hat{r}.$$

$$\frac{\mathrm{d}\bar{B}}{\bar{C}_0} = \frac{-A'\left(\frac{1}{2}\right)\hat{\nu}}{8 - 2A'\left(\frac{1}{2}\right)} - \frac{\bar{r}\mathrm{d}\bar{B}}{\bar{C}_0} - \frac{\hat{\nu}}{2},$$

$$\frac{\mathrm{d}\bar{B}}{\bar{C}_0} = \frac{-\hat{\nu}}{(1+\bar{r})\left[2 - \frac{1}{2}A'\left(\frac{1}{2}\right)\right]} < 0. \tag{58}$$

$$\frac{\mathrm{d}\bar{B}^*}{\bar{C}_0} = \frac{\hat{\nu}}{(1+\bar{r})\left[2 - \frac{1}{2}A'\left(\frac{1}{2}\right)\right]} > 0.$$

$$\widehat{\bar{C}^*} = \frac{\bar{r}\hat{\nu}}{(1+\bar{r})\left[2 - \frac{1}{2}A'\left(\frac{1}{2}\right)\right]} = -\hat{\bar{C}}. \tag{59}$$

$$\frac{w}{w^*} < \frac{A(z)}{1-\kappa} = \frac{a^*(z)}{(1-\kappa)a(z)},$$

$$\frac{w}{w^*} > (1-\kappa)A(z) = (1-\kappa)\frac{a^*(z)}{a(z)}.$$

$$\frac{w}{w^*} = \frac{A(z^{\mathrm{H}})}{1-\kappa}.$$

$$\frac{w}{w^*} = (1-\kappa)A(z^{\mathrm{F}}),$$

$$wa(z) < \frac{w^*a^*(z)}{1-\kappa},$$

$$w^*a^*(z) < \frac{wa(z)}{1-\kappa},$$

$$\begin{aligned} P &= \exp\left\{\int_0^{z^{\mathrm{H}}} \log\left[wa(z)\right] \mathrm{d}z + \int_{z^{\mathrm{H}}}^1 \log\left[\frac{w^*a^*(z)}{1-\kappa}\right] \mathrm{d}z\right\} \\ P^* &= \exp\left\{\int_0^{z^{\mathrm{F}}} \log\left[\frac{wa(z)}{1-\kappa}\right] \mathrm{d}z + \int_{z^{\mathrm{F}}}^1 \log\left[w^*a^*(z)\right] \mathrm{d}z\right\} \end{aligned} \tag{60}$$

$$\frac{P}{P^*} = \exp\left\{\int_{z^{\mathrm{F}}}^{z^{\mathrm{H}}} \log\left[\frac{wa(z)}{w^*a^*(z)}\right] \mathrm{d}z + \left[z^{\mathrm{F}} - \left(1 - z^{\mathrm{H}}\right)\right] \log(1-\kappa)\right\}$$

$$PC + P^*C^* = wL + w^*L^*. \tag{61}$$

$$\begin{aligned} wL &= z^{\mathrm{H}} PC + z^{\mathrm{F}} P^*C^* \\ &= z^{\mathrm{H}} PC + z^{\mathrm{F}} \left(wL + w^*L^* - PC \right). \end{aligned} \tag{62}$$

$$TB = wL - PC. \tag{63}$$

$$\frac{w}{w^*} = \left\{ \frac{-\left(z^{\mathrm{H}} - z^{\mathrm{F}}\right) TB}{\left[L^*/a^*(1)\right]} + z^{\mathrm{F}} \right\} \frac{L^*/L}{(1 - z^{\mathrm{H}})}, \tag{64}$$

$$(1-\kappa)A(z^{\mathrm{F}}) = A(z^{\mathrm{H}})/(1-\kappa).$$

$$A(z) = \exp(1-2z),$$

$$z^{\mathrm{H}} = z^{\mathrm{F}} - \log(1-\kappa). \tag{65}$$

$$\frac{w}{w^*} = \left\{ \frac{\log(1-\kappa)TB}{\left[L^*/a^*(1)\right]} + z^{\mathrm{F}} \right\} \frac{L^*/L}{\left[1+\log(1-\kappa) - z^{\mathrm{F}}\right]} \tag{66}$$

$$\frac{z^{\mathrm{F}}}{1-z^{\mathrm{H}}}\left(\frac{L^*}{L}\right)$$

$$\frac{w'a(z)}{1-\kappa} > w^{*\prime}a^*(z).$$

$$wa(z) < \frac{w^*a^*(z)}{1-\kappa}$$

$$P = \left[\gamma + (1-\gamma)w^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

$$C_{\mathrm{T},t} = \frac{(1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(w_s L_s - G_{\mathrm{T},s}\right)}{\sum_{s=t}^{\infty}\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t}{P_s}\right)^{\sigma-\theta}}$$

$$\bar{L} - L_s = C_{\mathrm{T},s}\left(\frac{1-\gamma}{\gamma}\right) w_s^{-\theta},$$

$$C_{\mathrm{T},t} = \frac{(1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[w_s\bar{L} - \left(\frac{P_s^{1-\theta}}{\gamma} - 1\right)C_{\mathrm{T},s} - G_{\mathrm{T},s}\right]}{\sum_{s=t}^{\infty}\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t}{P_s}\right)^{\sigma-\theta}}$$

$$0 = (1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(w_s\bar{L} - G_{\mathrm{T},s}\right)$$

$$-\sum_{s=t}^{\infty}\left(\frac{P_s^{1-\theta}}{\gamma}\right)\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t}{P_s}\right)^{\sigma-\theta} C_{\mathrm{T},t}$$

$$\sum_{s=t}^{\infty}\left(\frac{P_t^{1-\theta}}{\gamma}\right)\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_s}{P_t}\right)^{1-\theta}\left(\frac{P_t}{P_s}\right)^{\sigma-\theta}$$

$$=\left(\frac{P_t^{1-\theta}}{\gamma}\right)\sum_{s=t}^{\infty}\left[(1+r)^{s-t}\left(\frac{P_t}{P_s}\right)\right]^{\sigma-1}\beta^{\sigma(s-t)}.$$

$$C_{\mathrm{T},t} = \gamma \left(\frac{1}{P_t}\right)^{-\theta} \left\{ \frac{(1+r)Q_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[w_s \bar{L} - G_{\mathrm{T},s}\right]}{P_t \sum_{s=t}^{\infty} \left[(1+r)^{s-t} \left(\frac{P_t}{P_s}\right)\right]^{\sigma-1} \beta^{\sigma(s-t)}} \right\}$$

$$= \gamma \left(\frac{1}{P_t}\right)^{-\theta} C_t, \tag{67}$$

$$C_{\mathrm{T},t} = \gamma \left\{\frac{1}{P\left[w\left(r, A_t\right)\right]}\right\}^{1-\theta} \left[\frac{(1+r)Q_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[w\left(r, A_s\right) \bar{L} - G_{\mathrm{T},s}\right]}{\sum_{s=t}^{\infty} \left\{(1+r)^{s-t} \frac{P[w(r,A_t)]}{P[w(r,A_s)]}\right\}^{\sigma-1} \beta^{\sigma(s-t)}} \right]$$

$$\tag{68}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[p_s A_{\mathrm{N},s} K_{\mathrm{N},s}^{\alpha} L_{\mathrm{N},s}^{1-\alpha} - \frac{\chi}{2}\left(I_{\mathrm{N},s}^2 / K_{\mathrm{N},s}\right) - w_s L_{\mathrm{N},s} - I_{\mathrm{N},s}\right],$$

$$L_{\mathrm{N},s} = \left[\frac{(1-\alpha)p_s A_{\mathrm{N},s}}{w_s}\right]^{1/\alpha} K_{\mathrm{N},s}, \tag{69}$$

$$K_{\mathrm{N},s+1} - K_{\mathrm{N},s} = \frac{q_s - 1}{\chi} K_{\mathrm{N},s}, \tag{70}$$

$$\begin{aligned} q_{s+1} - q_s \\ &= r q_s - p_{s+1} A_{\mathrm{N},s} \alpha \left(\frac{L_{\mathrm{N},s+1}}{K_{\mathrm{N},s+1}}\right)^{1-\alpha} \\ &\quad - \frac{1}{2\chi}(q_{s+1} - 1)^2 \end{aligned} \tag{71}$$

$$C_{\mathrm{T},t} = \bar{C}_{\mathrm{T}}$$

$$= \frac{r}{1+r}\left\{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[Y_{\mathrm{T},s} - I_s - G_{\mathrm{T},s} - \frac{\chi}{2}\left(I_{\mathrm{N},s}^2/K_{\mathrm{N},s}\right)\right]\right\} \tag{72}$$

$$C_{\mathrm{N}} + G_{\mathrm{N}} = Y_{\mathrm{N}} = A_{\mathrm{N}} K_{\mathrm{N}}^{\alpha} L_{\mathrm{N}}^{1-\alpha}$$

$$\frac{(1-\gamma)\bar{C}_{\mathrm{T}}}{\gamma p} + \frac{\tilde{G}_{\mathrm{N}}}{p} = A_{\mathrm{N}} K_{\mathrm{N}}^{\alpha} L_{\mathrm{N}}^{1-\alpha}.$$

$$p = \frac{w^{1-\alpha}\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]^{\alpha}}{(1-\alpha)^{1-\alpha}A_{\mathrm{N}}K_{\mathrm{N}}^{\alpha}}, \tag{73}$$

$$L_{\mathrm{N}} = \frac{(1-\alpha)\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]}{w}. \tag{74}$$

$$\begin{aligned} q_{t+1} - q_t \\ &= rq_t - \frac{\alpha\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]}{K_{\mathrm{N},t}\left[1 + (q_t - 1)/\chi\right]} \\ &\quad - \frac{1}{2\chi}(q_{t+1} - 1)^2 \end{aligned} \tag{75}$$

$$\bar{q} = 1, \qquad \bar{K}_{\mathrm{N}} = \frac{\alpha\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]}{r}.$$

$$K_{\mathrm{N},t+1} - K_{\mathrm{N},t} = \frac{\bar{K}_{\mathrm{N}}}{\chi}\left(q_t - 1\right),$$

$$q_{t+1} - q_t = \left\{r + \frac{\alpha\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]}{\chi\bar{K}_{\mathrm{N}}}\right\}(q_t - 1)$$

$$+ \frac{\alpha\left[(1-\gamma)\bar{C}_{\mathrm{T}}/\gamma + \tilde{G}_{\mathrm{N}}\right]}{\bar{K}_{\mathrm{N}}^2}\left(K_{\mathrm{N},t} - \bar{K}_{\mathrm{N}}\right).$$

$$(Y_{\mathrm{T},t}\ \text{loss}) - \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} w\Delta L_{\mathrm{T}}$$

$$+\ (\text{present value of } I_{\mathrm{N}}, \text{ including installation losses}).$$

Table 4.1 Average Annual Labor Productivity Growth in Manufacturing, 1979–93

Country	**Productivity Growth (percent per year)**
Belgium	4.3
Canada	1.7
Denmark	1.5
France	2.8
Germany	1.9
Italy	4.1
Japan	3.8
Netherlands	2.6
Norway	2.3
Sweden	3.2
United Kingdom	4.1
United States	2.5

Source: Dean and Sherwood (1994). Data for Italy cover 1979–92 only.