3 The Life Cycle, Tax Policy, and the Current Account

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}. \qquad (1)$$

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}, \qquad (2)$$

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r},$$

$$S^{\mathrm{P}} = Y - T - C,$$

$$S^{\mathrm{G}} = T - G.$$

$$B_{t+1}^{\mathrm{P}} - B_t^{\mathrm{P}} = Y_t + r_t B_t^{\mathrm{P}} - T_t - C_t - I_t, \qquad (3)$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s)$$

$$= (1+r) B_t^{\mathrm{P}} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s) \, . \qquad (4)$$

$$B_{t+1}^{\mathrm{G}} - B_t^{\mathrm{G}} = T_t + r_t B_t^{\mathrm{G}} - G_t \tag{5}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} G_s$$

$$= (1+r) B_t^{\mathrm{G}} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} T_s, \tag{6}$$

$$B = B^{\mathrm{P}} + B^{\mathrm{G}}, \tag{7}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s+I_s)$$

$$=(1+r)\,B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s)\,. \tag{8}$$

$$U\left(c_t^{\mathrm{Y}},c_{t+1}^{\mathrm{O}}\right)=\log\left(c_t^{\mathrm{Y}}\right)+\beta\log\left(c_{t+1}^{\mathrm{O}}\right). \tag{9}$$

$$c_t^{\mathrm{Y}}+\frac{c_{t+1}^{\mathrm{O}}}{1+r}=y_t^{\mathrm{Y}}-\tau_t^{\mathrm{Y}}+\frac{y_{t+1}^{\mathrm{O}}-\tau_{t+1}^{\mathrm{O}}}{1+r} \tag{10}$$

$$c_{t+1}^{\mathrm{O}}=(1+r)\,\beta c_t^{\mathrm{Y}}. \tag{11}$$

$$c_t^{\mathrm{Y}} = \left(\frac{1}{1+\beta}\right)\left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}} + \frac{y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}}}{1+r}\right), \quad (12)$$

$$c_{t+1}^{\mathrm{O}} = (1+r)\left(\frac{\beta}{1+\beta}\right)\left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}} + \frac{y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}}}{1+r}\right). \quad (13)$$

$$C_t = c_t^{\mathrm{Y}} + c_t^{\mathrm{O}}. \quad (14)$$

$$B_{t+1}^{\mathrm{G}} - B_t^{\mathrm{G}} = \tau_t^{\mathrm{Y}} + \tau_t^{\mathrm{O}} + rB_t^{\mathrm{G}} - G_t. \quad (15)$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} G_s = (1+r)B_t^{\mathrm{G}}$$

$$+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(\tau_s^{\mathrm{Y}}+\tau_s^{\mathrm{O}}\right). \tag{16}$$

$$C = \left[\frac{1+(1+r)\beta}{1+\beta}\right]\left(y^{\mathrm{Y}} - \tau^{\mathrm{Y}} + \frac{y^{\mathrm{O}} - \tau^{\mathrm{O}}}{1+r}\right).$$

$$G = rB^{\mathrm{G}} + \tau^{\mathrm{Y}} + \tau^{\mathrm{O}},$$

$$C = \left[\frac{1 + (1+r)\beta}{1+\beta}\right]\left(y^{\mathrm{Y}} + \frac{y^{\mathrm{O}} - G - r\tau^{\mathrm{Y}} + rB^{\mathrm{G}}}{1+r}\right)$$

$$\begin{aligned} CA_t = B_{t+1} - B_t &= B^{\mathrm{P}}_{t+1} + B^{\mathrm{G}}_{t+1} - \left(B^{\mathrm{P}}_t + B^{\mathrm{G}}_t\right) \\ &= \left(B^{\mathrm{P}}_{t+1} - B^{\mathrm{P}}_t\right) + \left(B^{\mathrm{G}}_{t+1} - B^{\mathrm{G}}_t\right). \end{aligned} \tag{17}$$

$$S^{\mathrm{Y}}_t = B^{\mathrm{P}}_{t+1}. \tag{18}$$

$$S^{\mathrm{O}}_t = -S^{\mathrm{Y}}_{t-1} = -B^{\mathrm{P}}_t. \tag{19}$$

$$S_t^{\mathrm{P}} = S_t^{\mathrm{Y}} + S_t^{\mathrm{O}} = B_{t+1}^{\mathrm{P}} - B_t^{\mathrm{P}}, \tag{20}$$

$$B_{t+1} = B_{t+1}^{\mathrm{P}} + B_{t+1}^{\mathrm{G}} = S_t^{\mathrm{Y}} + B_{t+1}^{\mathrm{G}}. \tag{21}$$

$$\begin{aligned} S_t^{\mathrm{Y}} &= y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}} - c_t^{\mathrm{Y}} \\ &= \left(\frac{\beta}{1+\beta}\right)\left[\left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}}\right) - \left(y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}}\right)\right] = B_{t+1}^{\mathrm{P}} \end{aligned} \tag{22}$$

$$\begin{aligned} S_t^{\mathrm{P}} &= B_{t+1}^{\mathrm{P}} - B_t^{\mathrm{P}} = S_t^{\mathrm{Y}} - S_{t-1}^{\mathrm{Y}} \\ &= \left(\frac{\beta}{1+\beta}\right)\left[\Delta\left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}}\right) - \Delta\left(y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}}\right)\right] \end{aligned} \tag{23}$$

$$c_0^{\mathrm{O}\,\prime} = c_0^{\mathrm{O}} + \frac{d}{2}. \tag{24}$$

$$\begin{aligned} c_0^{\mathrm{Y}\prime} &= c_0^{\mathrm{Y}} + \frac{1}{1+\beta}\left(1 - \frac{r}{1+r}\right)\frac{d}{2} \\ &= c_0^{\mathrm{Y}} + \frac{1}{1+\beta}\left(\frac{1}{1+r}\right)\frac{d}{2}. \end{aligned} \tag{25}$$

$$c_0^{\mathrm{O}\prime} + c_0^{\mathrm{Y}\,\prime} - (c_0^{\mathrm{O}} + c_0^{\mathrm{Y}}) = \left[1 + \frac{1}{(1+\beta)(1+r)}\right]\frac{d}{2} \tag{26}$$

$$c_1^{O\prime} = c_1^O + (1+r)\frac{\beta}{1+\beta}\left(1 - \frac{r}{1+r}\right)\frac{d}{2}$$

$$= c_1^O + \left(\frac{\beta}{1+\beta}\right)\frac{d}{2} \tag{27}$$

$$-\left(1 + \frac{1}{1+r}\right)\left(\frac{rd}{2}\right) = -\left(\frac{2r + r^2}{1+r}\right)\left(\frac{d}{2}\right)$$

$$c_t^{Y\,\prime} = c_t^Y - \left(\frac{1}{1+\beta}\right)\left(\frac{2r + r^2}{1+r}\right)\left(\frac{d}{2}\right) \tag{28}$$

$$c_t^{\mathrm{O}\,\prime} = c_t^{\mathrm{O}} - \left(\frac{\beta}{1+\beta}\right)(2r + r^2)\left(\frac{d}{2}\right). \tag{29}$$

$$\begin{aligned} & c_1^{\mathrm{O}\,\prime} + c_1^{\mathrm{Y}\,\prime} - (c_1^{\mathrm{O}} + c_1^{\mathrm{Y}}) \\ & = \left[\left(\frac{\beta}{1+\beta}\right) - \left(\frac{1}{1+\beta}\right)\left(\frac{2r + r^2}{1+r}\right)\right]\left(\frac{d}{2}\right) \end{aligned} \tag{30}$$

$$\begin{aligned} & CA_0' - CA_0 = -\left[c_0^{\mathrm{O}\,\prime} + c_0^{\mathrm{Y}\,\prime} - (c_0^{\mathrm{O}} + c_0^{\mathrm{Y}})\right] \\ & = -\left[1 + \frac{1}{(1+\beta)(1+r)}\right]\frac{d}{2} \end{aligned} \tag{31}$$

$$CA_1' - CA_1 = r\left(CA_0' - CA_0\right) - \left[c_1^{\mathrm{O}\,\prime} + c_1^{\mathrm{Y}\,\prime} - (c_1^{\mathrm{O}} + c_1^{\mathrm{Y}})\right]$$

$$CA_1' - CA_1 = -\left(\frac{\beta}{1+\beta}\right)(1+r)\left(\frac{d}{2}\right) \qquad (32)$$

$$CA/Y = \underset{(4.06)}{-3.55} + \underset{(0.33)}{0.78}(T-G)/Y, \qquad R^2 = 0.24.$$

$$u'(c_t^{\mathrm{Y}}) = (1 + r_{t+1})\beta u'(c_{t+1}^{\mathrm{O}}).$$

$$u'(c_t) = \beta \mathrm{E}_t \left\{ (1 + r_{t+1}) u'(c_{t+1}) \right\}$$

$$CA_0{}' - CA_0 = \left(\frac{\beta}{1+\beta} \right) \mathrm{d}y,$$

$$CA_1{}' - CA_1 = -\left(\frac{\beta}{1+\beta} \right) \mathrm{d}y$$

$$S_t^{\mathrm{P}} = B_{t+1}^{\mathrm{P}} - B_t^{\mathrm{P}} = S_t^{\mathrm{Y}} - S_{t-1}^{\mathrm{Y}}$$

$$= \left(\frac{\beta}{1+\beta} \right) \left[\Delta \left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}} \right) - \Delta \left(y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}} \right) \right] \quad (23)$$

$$S_t^{\mathrm{P}} = \left(\frac{\beta}{1+\beta}\right)\{\underbrace{\left(y_t^{\mathrm{Y}} - \tau_t^{\mathrm{Y}}\right) - \left(y_{t+1}^{\mathrm{O}} - \tau_{t+1}^{\mathrm{O}}\right)}_{(1+\beta)S_t^{\mathrm{Y}}/\beta}$$

$$- \underbrace{\left[\left(y_{t-1}^{\mathrm{Y}} - \tau_{t-1}^{\mathrm{Y}}\right) - \left(y_t^{\mathrm{O}} - \tau_t^{\mathrm{O}}\right)\right]}_{(1+\beta)S_{t-1}^{\mathrm{Y}}/\beta}\}.$$

$$y_{t+1}^{\mathrm{O}} = (1+e)y_t^{\mathrm{Y}},$$

$$y_{t+1}^{\mathrm{Y}} = (1+g)y_t^{\mathrm{Y}}.$$

$$\frac{S_t^{\mathrm{P}}}{Y_t} = -\frac{\beta}{1+\beta}\left(\frac{eg}{2+e+g}\right).$$

$$\frac{\mathrm{d}(S_t^{\mathrm{P}}/Y_t)}{\mathrm{d}e} = -\frac{\beta}{1+\beta}\left[\frac{g(2+g)}{(2+e+g)^2}\right] < 0.$$

$$\frac{\mathrm{d}(S_t^{\mathrm{P}}/Y_t)}{\mathrm{d}g} = -\frac{\beta}{1+\beta}\left[\frac{e(2+e)}{(2+e+g)^2}\right],$$

$$\frac{S_t^{\mathrm{P}}}{Y_t} = \frac{\left(N_t - N_{t-1}\right) s^{\mathrm{Y}}}{N_t y^{\mathrm{Y}} + N_{t-1} y^{\mathrm{O}}} = \frac{n s^{\mathrm{Y}}}{(1+n) y^{\mathrm{Y}} + y^{\mathrm{O}}}, \qquad (33)$$

$$\frac{\mathrm{d}\left(S^{\mathrm{P}}/Y\right)}{\mathrm{d}n} = \frac{s^{\mathrm{Y}}\left(y^{\mathrm{Y}} + y^{\mathrm{O}}\right)}{\left[(1+n)y^{\mathrm{Y}} + y^{\mathrm{O}}\right]^2} > 0.$$

$$S^{\mathrm{P}}/Y = \underset{(1.3)}{4.5} + \underset{(0.25)}{1.42}z.$$

$$S/Y = \underset{(1.90)}{7.93} + \underset{(0.47)}{0.99}z, \qquad R^2 = 0.03.$$

$$Y_t = A_t F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{34}$$

$$N_t = (1+n)N_{t-1}. \tag{35}$$

$$r = A_t F_K(K_t, L_t) = \alpha A_t k_t^{\alpha-1}, \tag{36}$$

$$w_t = A_t F_L(K_t, L_t) = (1-\alpha) A_t k_t^{\alpha}, \tag{37}$$

$$K(r, A_t, N_t) = L_t k(r, A_t) = N_t k(r, A_t)$$

$$K(r, A_t, N_t) = N_t k(r, A_t) = N_t \left(\frac{\alpha A_t}{r}\right)^{1/1-\alpha} \tag{38}$$

$$w_t = (1-\alpha)A_t k(r, A_t)^{\alpha} = (1-\alpha)A_t \left(\frac{\alpha A_t}{r}\right)^{\alpha/1-\alpha} \tag{39}$$

$$S_t^{\mathrm{Y}} = B_{t+1}^{\mathrm{P}} + K_{t+1} = B_{t+1} + K_{t+1}. \tag{40}$$

$$s_t^{\mathrm{Y}} = (1+n)(b_{t+1} + k_{t+1}), \tag{41}$$

$$\bar{b} = \frac{\bar{s}^{\mathrm{Y}}}{1+n} - \bar{k}. \tag{42}$$

$$\frac{S_t^{\mathrm{Y}} + S_t^{\mathrm{O}}}{N_t + N_{t-1}} = \left(\frac{1+n}{2+n}\right)\bar{s}^{\mathrm{Y}} + \left(\frac{1}{2+n}\right)\bar{s}^{\mathrm{O}}.$$

$$\frac{K_{t+1} - K_t}{N_t + N_{t-1}} = \frac{(1+n)n}{2+n}\bar{k}.$$

$$A_{t+1} = (1+g)^{1-\alpha} A_t, \tag{43}$$

$$\frac{\overline{K}}{Y} = \frac{\alpha}{r}$$

$$\frac{\overline{I}}{Y} = \left(\frac{N_{t+1} A_{t+1}^{\frac{1}{1-\alpha}}}{N_t A_t^{\frac{1}{1-\alpha}}} \right) \frac{\overline{K}}{Y} - \frac{\overline{K}}{Y} = (n + g + ng) \frac{\alpha}{r}. \tag{44}$$

$$c_t^{\mathrm{Y}} = \frac{w_t}{1+\beta}, \qquad c_{t+1}^{\mathrm{O}} = \frac{(1+r)\beta w_t}{1+\beta}. \tag{45}$$

$$s_t^{\mathrm{Y}} = w_t - \frac{w_t}{1+\beta} = \frac{\beta w_t}{1+\beta}$$

$$= \frac{\beta(1-\alpha)A_t^{\frac{1}{1-\alpha}}}{1+\beta}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\frac{N_t s_t^{\mathrm{Y}} + N_{t-1}s_t^{\mathrm{O}}}{Y_t} = \frac{\overline{S}}{Y}$$

$$= \frac{\beta(1-\alpha)}{1+\beta}\left[1 - \frac{1}{(1+n)(1+g)}\right] \tag{46}$$

$$\frac{\overline{B}}{Y} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)(1+g)} - \frac{\alpha}{r}.$$

$$\frac{\overline{CA}}{Y} = \frac{\overline{S}}{Y} - \frac{\overline{I}}{Y} = (n + g + ng)\frac{\overline{B}}{Y}.$$

$$\underset{(0.02)\ (0.07)}{I/Y = 0.04 + 0.89 S/Y}, \qquad R^2 = 0.91\ .$$

$$\underset{(0.02)\ (0.09)}{I/Y = 0.09 + 0.62 S/Y}, \qquad R^2 = 0.69\ .$$

$$U = (1 + \beta)\log(w) + \beta\log(1 + r)$$

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \frac{1+\beta}{w}\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right) + \frac{\beta}{1+r},$$

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \frac{-(1+\beta)}{w}k + \frac{\beta}{1+r}$$

$$= -\beta + \frac{\beta}{1+r} = \frac{-\beta r}{1+r} < 0 \tag{47}$$

$$-\,k\mathrm{d}r + \frac{\beta w \mathrm{d}r}{(1+\beta)(1+r)}$$

$$= -k\mathrm{d}r + \frac{k\mathrm{d}r}{1+r} = \frac{-rk\mathrm{d}r}{1+r}$$

$$\frac{-rk\mathrm{d}r}{(1+r)}\left[1+\left(\frac{1}{1+r}\right)+\left(\frac{1}{1+r}\right)^2+\cdots\right]=-k\mathrm{d}r$$

$$\frac{\mathrm{d}U}{\mathrm{d}r}=\frac{-(1+\beta)}{w}k+\frac{\beta}{1+r}=\frac{-\beta k}{k+b}+\frac{\beta}{1+r}$$

$$-k\mathrm{d}r+\frac{(k+b)\mathrm{d}r}{1+r},$$

$$s_t^{\mathrm{Y}}=\frac{\beta}{1+\beta}\left(w_t-\tau_t^{\mathrm{Y}}\right). \tag{48}$$

$$w_t = (1-\alpha)k_t^{\alpha}. \tag{49}$$

$$\alpha k_t^{\alpha-1} = \alpha k_t^{*\alpha-1} = r_t = \alpha (k_t^{\mathrm{W}})^{\alpha-1}, \tag{50}$$

$$K_{t+1} + K_{t+1}^{*} = N_t s_t^{\mathrm{Y}} + N_t^{*} s_t^{\mathrm{Y}*}. \tag{51}$$

$$L_t = N_t, \qquad L_t^{*} = N_t^{*}.$$

$$s_t^{\mathrm{Y}} = \frac{\beta(1-\alpha)}{1+\beta}(k_t^{\mathrm{W}})^{\alpha},$$

$$K_{t+1} + K_{t+1}^* = \frac{\beta(1-\alpha)}{1+\beta}\left(N_t + N_t^*\right)(k_t^{\mathrm{W}})^{\alpha}$$

$$\frac{K_{t+1} + K_{t+1}^*}{N_t + N_t^*} = (1+n)\frac{K_{t+1} + K_{t+1}^*}{N_{t+1} + N_{t+1}^*}$$

$$= (1+n)k_{t+1}^{\mathrm{W}}$$

$$k_{t+1}^{\mathrm{W}} = \frac{\beta(1-\alpha)}{(1+n)(1+\beta)}(k_t^{\mathrm{W}})^{\alpha} \equiv \Psi(k_t^{\mathrm{W}}). \qquad (52)$$

$$\bar{k}^{\mathrm{W}} = \left[\frac{\beta(1-\alpha)}{(1+n)(1+\beta)}\right]^{\frac{1}{1-\alpha}}.$$

$$\alpha(\bar{k}^{\mathrm{W}})^{\alpha-1} = \bar{r} = \frac{\alpha(1+n)(1+\beta)}{\beta(1-\alpha)}. \tag{53}$$

$$B_{t+1}^{\mathrm{G}} = (1+r_t)B_t^{\mathrm{G}} + N_t\tau_t^{\mathrm{Y}},$$

$$\bar{d} = \frac{-B^{G}_{t+1}}{N_{t+1}} = (1 + r_t)\frac{N_t}{N_{t+1}}\bar{d} - \frac{N_t}{N_{t+1}}\tau^{Y}_{t}$$

$$= \frac{(1 + r_t)\bar{d} - \tau^{Y}_{t}}{1 + n}$$

$$\tau^{Y}_{t} = (r_t - n)\bar{d}. \tag{54}$$

$$s^{Y}_{t} = \frac{\beta}{1 + \beta}\left[w_t - (r_t - n)\bar{d}\right].$$

$$s^{Y}_{t} = \frac{\beta}{1 + \beta}\left\{(1 - \alpha)\left(k^{W}_{t}\right)^{\alpha} - \left[\alpha\left(k^{W}_{t}\right)^{\alpha - 1} - n\right]\bar{d}\right\} \tag{55}$$

$$K_{t+1} + K^*_{t+1} - B^{\mathrm{G}}_{t+1} = N_t s^{\mathrm{Y}}_t + N^*_t s^{\mathrm{Y}*}_t.$$

$$k^{\mathrm{W}}_{t+1} = \frac{\beta\left\{(1-\alpha)(k^{\mathrm{W}}_t)^{\alpha} - x\left[\alpha(k^{\mathrm{W}}_t)^{\alpha-1} - n\right]\bar{d}\right\}}{(1+n)(1+\beta)}$$

$$- x\bar{d} \equiv \Psi\left(k^{\mathrm{W}}_t, \bar{d}\right)$$

$$x \equiv \frac{N_t}{N_t + N^*_t}$$

$$U_t = u(C_t) + \beta U_{t+1}, \tag{56}$$

$$(1+r)H_t + Y_t - T_t = C_t + H_{t+1}, \tag{57}$$

$$H_{t+1} \geq 0. \tag{58}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s$$

$$= (1+r)H_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - T_s), \tag{59}$$

$$U_{t+1} = u(C_{t+1}) + \beta U_{t+2}.$$

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 U_{t+2}.$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) + \lim_{s \to \infty} \beta^{s-t} U_s.$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \tag{60}$$

$$\frac{\partial U^{\mathrm{P}}}{\partial a_1} > 0, \qquad \frac{\partial U^{\mathrm{C}}}{\partial a_1} \leq 0.$$

$$U_t^v = \sum_{s=t}^{\infty} \beta^{s-t} \log(c_s^v), \tag{61}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} c_s^v = (1+r) b_t^{\mathrm{P},v} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(y_s^v - \tau_s^v\right) \tag{62}$$

$$b_v^{\mathrm{P},v} = 0. \tag{63}$$

$$c_t^v = (1-\beta)\left[(1+r)b_t^{\mathrm{P},v} + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(y_s^v - \tau_s^v\right)\right] \tag{64}$$

$$c_t = \frac{c_t^0 + nc_t^1 + n(1+n)c_t^2 + \cdots + n(1+n)^{t-1}c_t^t}{(1+n)^t} \tag{65}$$

$$c_t = (1-\beta)\left[(1+r)b_t^{\mathrm{P}} + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(y_s - \tau_s)\right] \tag{66}$$

$$b_{t+1}^{\mathrm{P},v} = (1+r)b_t^{\mathrm{P},v} + y_t^v - \tau_t^v - c_t^v,$$

$$\frac{b_{t+1}^{\mathrm{P},0} + nb_{t+1}^{\mathrm{P},1} + \cdots + n(1+n)^{t-1}b_{t+1}^{\mathrm{P},t}}{(1+n)^t}$$

$$= (1+r)b_t^{\mathrm{P}} + y_t - \tau_t - c_t,$$

$$(1+n)\frac{\left[b_{t+1}^{\mathrm{P},0} + nb_{t+1}^{\mathrm{P},1} + \cdots + n(1+n)^{t-1}b_{t+1}^{\mathrm{P},t} + n(1+n)^t b_{t+1}^{\mathrm{P},t+1}\right]}{(1+n)^{t+1}}$$

$$= (1+n)b_{t+1}^{\mathrm{P}}$$

$$b_{t+1}^{\mathrm{P}} = \frac{(1+r)b_t^{\mathrm{P}} + y_t - \tau_t - c_t}{1+n} \tag{67}$$

$$b_{t+1}^{\mathrm{P}} = \left[\frac{(1+r)\beta}{1+n}\right] b_t^{\mathrm{P}} + \left[\frac{y_t - \tau_t - (1-\beta)\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(y_s - \tau_s)}{1+n}\right] \tag{68}$$

$$b_{t+1} = \left[\frac{(1+r)\beta}{1+n}\right] b_t + \left[\frac{(1+r)\beta - 1}{r(1+n)}\right] \bar{y}. \tag{69}$$

$$\bar{b} = \left[\frac{(1+r)\beta - 1}{(1+n) - (1+r)\beta}\right]\frac{\bar{y}}{r}. \tag{70}$$

$$b_{t+1} = \frac{(1+r)b_t + y_t - c_t}{1+n}.$$

$$\bar{c} = (r-n)\bar{b} + \bar{y}. \tag{71}$$

$$y_s = \begin{cases} \bar{y}' & (s = t), \\ \bar{y} & (s > t). \end{cases} \tag{72}$$

$$b_{t+1} = \left[\frac{(1+r)\beta}{1+n}\right]\bar{b}$$

$$+ \left\{\frac{\bar{y}' - (1-\beta)\left[\bar{y}' + \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\bar{y}\right]}{1+n}\right\}$$

$$= \left[\frac{(1+r)\beta}{1+n}\right]\bar{b} + \left[\frac{(1+r)\beta - 1}{r(1+n)}\right]\bar{y} + \frac{\beta}{1+n}(\bar{y}' - \bar{y})$$

$$= \bar{b} + \frac{\beta}{1+n}(\bar{y}' - \bar{y}) > \bar{b},$$

$$y_{t+1} = (1+g)y_t,$$

$$c_t = (1-\beta)\left[(1+r)b_t + \left(\frac{1+r}{r-g}\right)y_t\right]$$

$$b_{t+1} = \left[\frac{(1+r)\beta}{1+n}\right]b_t + \left[\frac{(1+r)\beta-(1+g)}{(1+n)(r-g)}\right]y_t.$$

$$\frac{b_{t+1}}{y_{t+1}} = \left[\frac{(1+r)\beta}{(1+n)(1+g)}\right]\frac{b_t}{y_t} + \left[\frac{(1+r)\beta-(1+g)}{(1+n)(1+g)(r-g)}\right] \quad (73)$$

$$b_{t+1} = \frac{b_t}{1+n},$$

$$b^{G}_{t+1} = \frac{(1+r)\, b^{G}_{t} + \tau_t - g_t}{1+n}, \tag{74}$$

$$\tau_s = (r-n)\bar{d} + g_s. \tag{75}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tau_s = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (r-n)\bar{d} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} g_s$$

$$c_t = (1-\beta)\left[(1+r)(b_t+\bar{d}) + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(y_s - g_s) - \frac{(1+r)(r-n)}{r}\bar{d}\right]$$

$$= (1-\beta)\left[(1+r)\left(b_t + \frac{n\bar{d}}{r}\right) + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(y_s - g_s)\right].$$

$$Y = F(K, EL), \tag{76}$$

$$E_{t+1} = (1+g)E_t,$$

$$E_{t+1}L_{t+1} = (1+n)E_t(1+g)L_t = (1+z)E_tL_t,$$

$$1 + z \equiv (1 + n)(1 + g).$$

$$y^{\mathrm{E}} = F(K/EL, 1) \equiv f(k^{\mathrm{E}}).$$

$$K_{t+1} - K_t = F(K_t, E_t L_t) - C_t,$$

$$k_{t+1}^{\mathrm{E}} = \frac{k_t^{\mathrm{E}} + f(k_t^{\mathrm{E}}) - c_t^{\mathrm{E}}}{1 + z},$$

$$\bar{c}^{\mathrm{E}} = f(\bar{k}^{\mathrm{E}}) - z\bar{k}^{\mathrm{E}}. \tag{77}$$

$$\frac{\mathrm{d}\,\bar{c}^{\mathrm{E}}}{\mathrm{d}\,\bar{k}^{\mathrm{E}}} = 0 \Longleftrightarrow f'(\bar{k}^{\mathrm{E}}) = \bar{r} = z. \tag{78}$$

$$B^{\mathrm{G}}_{t+1} = -(1+r)^t D.$$

$$\frac{p_{t+1}}{p_t} = 1 + r,$$

$$\frac{rK}{Y} > \frac{zK}{Y}.$$

Table 3.1 Growth and Saving in the Seven Largest Industrial Countries, 1960–87

Period	GNP Growth Rate	Net Private Saving Rate
1960–70	4.9	12.3
1971–80	3.3	11.1
1981–87	2.6	9.1

Source: Guiso, Jappelli, and Terlizzese (1992). Growth rates are a simple average of period average growth rates for Canada, France, Italy, West Germany, Japan, the United Kingdom, and the United States, expressed in percent per year. Saving rates are a simple average of period average ratios of inflation-adjusted net private saving to net national product, expressed in percent.

Table 3.2 Government Saving in the Main Industrial Countries

Country	1960s	1970s	1980s
Canada	3.6	2.7	−1.6
France	n.a.	3.6	1.3
West Germany	6.2	3.9	2.0
Italy	2.1	−5.6	−6.7
Japan	6.2	4.8	4.6
United Kingdom	3.6	2.6	0.1
United States	2.0	0.4	−2.1

Source: Shafer, Elmeskov, and Tease (1992). Government budget surpluses are expressed as a percent of GNP.