2 Dynamics of Small Open Economies

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \ldots + \beta^T u(C_{t+T}) = \sum_{s=t}^{t+T} \beta^{s-t} u(C_s). \qquad (1)$$

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t \qquad (2)$$

$$(1+r)B_t = C_t + G_t + I_t - Y_t + B_{t+1}. \qquad (3)$$

$$B_{t+1} = \frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r},$$

$$(1+r)B_t = C_t + G_t + I_t - Y_t + \frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r}.$$

$$\frac{B_{t+2}}{1+r} = \frac{C_{t+2} + G_{t+2} + I_{t+2} - Y_{t+2}}{(1+r)^2} + \frac{B_{t+3}}{(1+r)^2},$$

$$\sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s) + \left(\frac{1}{1+r}\right)^{T} B_{t+T+1}$$

$$= (1+r)B_t + \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s). \qquad (4)$$

$$B_{s+1} - B_s = rB_s + A_s F(K_s)$$
$$- C_s - (K_{s+1} - K_s) - G_s$$

$$U_t = \sum_{s=t}^{t+T} \beta^{s-t} u \Big[(1+r)B_s - B_{s+1}$$
$$+ A_s F(K_s) - (K_{s+1} - K_s) - G_s\Big]$$

$$u'(C_s) = (1 + r)\beta u'(C_{s+1}), \tag{5}$$

$$A_{s+1}F'(K_{s+1}) = r. \tag{6}$$

$$B_{t+T+1} = 0 \tag{7}$$

$$\sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t}(C_s+I_s)$$

$$=(1+r)B_t+\sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s) \quad (8)$$

$$C_t=\left[\frac{1}{1-(1+r)^{-(T+1)}}\right]\left(\frac{r}{1+r}\right)$$

$$\cdot\left[(1+r)B_t+\sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s-I_s)\right] \quad (9)$$

$$C_t=\frac{r}{1+r}\left[(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s-I_s)\right] \quad (10)$$

$$U_t = \lim_{T \to \infty} \left[u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + ... \right]$$

$$= \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \tag{11}$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \left[(1+r) B_s - B_{s+1} \right.$$

$$\left. + A_s F(K_s) - (K_{s+1} - K_s) - G_s \right]$$

$$
\begin{aligned}
B_{t+T+1} &= (1+r)B_{t+T} + \bar{Y} - \bar{C} \\
&= (1+r)\left[(1+r)B_{t+T-1} + \bar{Y} - \bar{C}\right] + \bar{Y} - \bar{C} \\
&= ... = (1+r)^{T+1}B_t + \sum_{s=0}^{T}(1+r)^s\left(\bar{Y} - \bar{C}\right) \\
&= (1+r)^{T+1}B_t - \frac{1-(1+r)^{T+1}}{r}\left(\bar{Y} - \bar{C}\right) \\
&= B_t + \left(rB_t + \bar{Y} - \bar{C}\right)\left[\frac{(1+r)^{T+1}-1}{r}\right] \qquad (12)
\end{aligned}
$$

$$
\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0. \qquad (13)
$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s+I_s)$$

$$= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s) \tag{14}$$

$$-(1+r)B_t$$

$$=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-C_s-I_s-G_s)$$

$$B_{s+1}-B_s = gB_s = rB_s + TB_s,$$

$$\frac{TB_s}{Y_s} = \frac{-(r-g)B_s}{Y_s} = \frac{-B_s}{Y_s/(r-g)}.$$

$$C_t = \bar{C} = rB_t + \bar{Y}.$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(\bar{C}+I_s)$$

$$= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)$$

$$\bar{C} = C_t = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$$

$$C_{s+1} = (1+r)^{\sigma}\beta^{\sigma}C_s \tag{15}$$

$$C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)}{\sum_{s=t}^{\infty}\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}}$$

$$C_t = \frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right] \tag{16}$$

$$U_{t+1} = \sum_{s=t+1}^{\infty} \beta^{s-(t+1)} u(C_s),$$

$$U_t = (1+\gamma)u(C_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} u(C_s),$$

$$U_{t+1} = (1+\gamma)u(C_{t+1}) + \sum_{s=t+2}^{\infty} \beta^{s-(t+1)} u(C_s)$$

$$(1+\gamma)u(\bar{C}_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} u(C_s)$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

$$\tilde{X}_t\equiv\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s. \tag{17}$$

$$\begin{aligned}CA_t&=B_{t+1}-B_t\\&=(Y_t-\tilde{Y}_t)-(I_t-\tilde{I}_t)-(G_t-\tilde{G}_t)\end{aligned} \tag{18}$$

$$W_t\equiv(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-I_s-G_s) \tag{19}$$

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t)$$

$$- (G_t - \tilde{G}_t) - \frac{\vartheta}{1+r} W_t \quad (20)$$

$$R_{t,s} = \frac{1}{\prod_{v=t+1}^{s} (1 + r_v)} \quad (21)$$

$$B_{s+1} - B_s = Y_s + r_s B_s - C_s - G_s - I_s \quad (22)$$

$$\sum_{s=t}^{\infty} R_{t,s}(C_s + I_s)$$

$$= (1 + r_t)B_t + \sum_{s=t}^{\infty} R_{t,s}(Y_s - G_s) \tag{23}$$

$$\lim_{T\to\infty} R_{t,t+T}B_{t+T+1} = 0$$

$$u'(C_s) = (1 + r_{s+1})\beta u'(C_{s+1}) \tag{24}$$

$$A_{s+1}F'(K_{s+1}) = r_{s+1}$$

$$C_t = \frac{(1 + r_t)B_t + \sum_{s=t}^{\infty} R_{t,s}(Y_s - I_s - G_s)}{\sum_{s=t}^{\infty} R_{t,s}\left[R_{t,s}^{-\sigma}\beta^{\sigma(s-t)}\right]} \tag{25}$$

$$\sum_{s=t}^{\infty} R_{t,s}\tilde{X}_t = \sum_{s=t}^{\infty} R_{t,s}X_s$$

$$\tilde{X}_t \equiv \frac{\sum_{s=t}^{\infty} R_{t,s}X_s}{\sum_{s=t}^{\infty} R_{t,s}}$$

$$\tilde{\Gamma}_t \equiv \frac{\sum_{s=t}^{\infty} R_{t,s}\left[R_{t,s}^{-\sigma}\beta^{\sigma(s-t)}\right]}{\sum_{s=t}^{\infty} R_{t,s}}.$$

$$CA_t = (r_t - \tilde{r}_t)B_t + (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)$$
$$+ \left(\frac{\tilde{\Gamma}_t - 1}{\tilde{\Gamma}_t}\right)\left(\tilde{r}_t B_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t\right) \qquad (26)$$

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s + I_s)$$

$$= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)$$

$$U_t = \mathrm{E}_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}u\left[(1+r)B_s - B_{s+1} + Y_s - G_s - I_s\right]\right\} \quad (27)$$

$$\mathrm{E}_t\left\{u'(C_s)\right\} = (1+r)\beta\mathrm{E}_t\left\{u'(C_{s+1})\right\} \quad (28)$$

$$u'(C_t) = (1+r)\beta \mathrm{E}_t \left\{ u'(C_{t+1}) \right\}. \tag{29}$$

$$u(C) = C - \frac{a_0}{2}C^2, \qquad a_0 > 0. \tag{30}$$

$$\mathrm{E}_t C_{t+1} = C_t; \tag{31}$$

$$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (C_s + I_s) \right\}$$
$$= \mathrm{E}_t \left\{ (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s) \right\}.$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_t$$

$$=\mathrm{E}_t\left\{(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s-I_s)\right\}$$

$$C_t=\frac{r}{1+r}\left[(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathrm{E}_t\left\{Y_s-G_s-I_s\right\}\right] \tag{32}$$

$$Y_{t+1}-\bar{Y}=\rho\left(Y_t-\bar{Y}\right)+\epsilon_{t+1}, \tag{33}$$

$$\mathrm{E}_t\left\{Y_s - \bar{Y}\right\} = \rho^{s-t}(Y_t - \bar{Y}), \tag{34}$$

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathrm{E}_t\left\{Y_s - \bar{Y}\right\}$$

$$C_t = rB_t + \bar{Y} + \frac{r(Y_t - \bar{Y})}{1+r-\rho}. \tag{35}$$

$$Y_t = \bar{Y} + \sum_{s=-\infty}^{t}\rho^{t-s}\epsilon_s, \tag{36}$$

$$C_t = rB_t + \bar{Y}$$

$$+ \frac{r\rho}{1+r-\rho}(Y_{t-1} - \bar{Y}) + \frac{r}{1+r-\rho}\epsilon_t$$

$$CA_t = \rho\left(\frac{1-\rho}{1+r-\rho}\right)(Y_{t-1} - \bar{Y}) + \left(\frac{1-\rho}{1+r-\rho}\right)\epsilon_t \quad (37)$$

$$\mathrm{E}_{t-1}\left\{Y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s\right\}$$

$$= \rho(Y_{t-1} - \bar{Y}) - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \rho^{s-t+1}(Y_{t-1} - \bar{Y})$$

$$= \rho\left(\frac{1-\rho}{1+r-\rho}\right)(Y_{t-1} - \bar{Y}).$$

$$Y_{t+1} - Y_t = \rho \left(Y_t - Y_{t-1} \right) + \epsilon_{t+1} \tag{38}$$

$$Y_t = Y_{t-1} + \sum_{s=-\infty}^{t} \rho^{t-s} \epsilon_s. \tag{39}$$

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u \left[(1+r) B_s - B_{s+1} \right.\right.$$

$$\left.\left. + A_s F(K_s) - (K_{s+1} - K_s) - G_s \right] \right\}$$

$$u'(C_t) = \mathrm{E}_t \left\{ \left[1 + A_{t+1} F'(K_{t+1}) \right] \beta u'(C_{t+1}) \right\}.$$

$$1 = \mathrm{E}_t \left\{ \left[1 + A_{t+1} F'(K_{t+1})\right] \frac{\beta u'(C_{t+1})}{u'(C_t)} \right\}$$

$$= \mathrm{E}_t \left\{1 + A_{t+1} F'(K_{t+1})\right\} \mathrm{E}_t \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)} \right\}$$

$$+ \mathrm{Cov}_t \left\{ A_{t+1} F'(K_{t+1}), \frac{\beta u'(C_{t+1})}{u'(C_t)} \right\}$$

$$\mathrm{E}_t \left\{A_{t+1} F'(K_{t+1})\right\}$$

$$= r - \mathrm{Cov}_t \left\{ A_{t+1} F'(K_{t+1}), \frac{u'(C_{t+1})}{u'(C_t)} \right\} \tag{40}$$

$$A_{t+1} - \bar{A} = \rho(A_t - \bar{A}) + \epsilon_{t+1}, \tag{41}$$

$$\Delta I_t = a_0 + a_1 \Delta A_t^{\mathrm{C}} + a_2 \Delta A_t^{\mathrm{W}} + a_3 I_{t-1},$$

$$\Delta CA_t = b_0 + b_1 \Delta A_t^{\mathrm{C}} + b_2 \Delta A_t^{\mathrm{W}} + b_3 I_{t-1},$$

$$\begin{aligned} CA_t &= B_{t+1} - B_t \\ &= (Y_t - \mathrm{E}_t \tilde{Y}_t) - (I_t - \mathrm{E}_t \tilde{I}_t) - (G_t - \mathrm{E}_t \tilde{G}_t) \end{aligned} \tag{42}$$

$$\mathrm{E}_t \tilde{X}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \mathrm{E}_t X_s$$

$$Z \equiv Y - G - I$$

$$CA_t = -\sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \mathrm{E}_t \Delta Z_s \tag{43}$$

$$\begin{bmatrix} \Delta Z_s \\ CA_s \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} \Delta Z_{s-1} \\ CA_{s-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1s} \\ \epsilon_{2s} \end{bmatrix} \tag{44}$$

$$\mathrm{E}_t \begin{bmatrix} \Delta Z_s \\ CA_s \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{s-t} \begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix}$$

$$\mathrm{E}_t \Delta Z_s = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{s-t} \begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix}$$

$$\widehat{CA}_t = -\begin{bmatrix} 1 & 0 \end{bmatrix} \left(\frac{1}{1+r}\Psi\right) \left(\mathbf{I} - \frac{1}{1+r}\Psi\right)^{-1} \begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix}$$

$$\equiv \begin{bmatrix} \Phi_{\Delta Z} & \Phi_{CA} \end{bmatrix} \begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix} \tag{45}$$

$$\begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix} = \begin{bmatrix} 0.20 & -0.09 \\ -0.03 & 0.83 \end{bmatrix} \begin{bmatrix} \Delta Z_{t-1} \\ CA_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\gamma \log C_s + (1-\gamma) \log D_s\right] \tag{46}$$

$$B_{s+1} - B_s = r_s B_s + Y_s - C_s - p_s[D_s - (1-\delta)D_{s-1}] - (K_{s+1} - K_s) - G_s$$

$$C_{s+1} = (1 + r_{s+1})\beta C_s,$$

$$\frac{\gamma p_s}{C_s} = \frac{1-\gamma}{D_s} + \beta(1-\delta)\frac{\gamma p_{s+1}}{C_{s+1}}$$

$$\frac{(1-\gamma)C_s}{\gamma D_s} = p_s - \frac{1-\delta}{1+r_{s+1}} p_{s+1} \equiv \iota_s \tag{47}$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + \iota_s D_s) = (1+r)B_t$$

$$+ (1-\delta) p_t D_{t-1} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s) \qquad (48)$$

$$C_t = \frac{\gamma r}{1+r} \Bigg[(1+r)B_t + (1-\delta) p_t D_{t-1}$$

$$+ \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s) \Bigg] \qquad (49)$$

$$D_t = \frac{(1-\gamma)r}{\iota_t(1+r)}\left[(1+r)B_t + (1-\delta)p_t D_{t-1}\right.$$

$$\left. + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$$

$$p = \left(\frac{1+r}{r+\delta}\right)\left(\frac{1-\gamma}{\gamma}\right)\frac{C_s}{D_s}. \tag{50}$$

$$CA_t = B_{t+1} - B_t = rB_t + Z_t - \frac{C_t}{\gamma}$$

$$+ \frac{(1-\gamma)C_t}{\gamma} - p[D_t - (1-\delta)D_{t-1}].$$

$$CA_t = \left\{ Z_t - \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Z_s \right\}$$

$$- \frac{r}{1+r}(1-\delta)pD_{t-1} + \frac{(1-\gamma)C_t}{\gamma} - pD_t + (1-\delta)pD_{t-1},$$

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t)$$

$$- (G_t - \tilde{G}_t) + (\iota - p)\Delta D_t \qquad (51)$$

$$B_{s+1} - B_s + V_s x_{s+1} - V_{s-1} x_s = rB_s + d_s x_s$$

$$+ (V_s - V_{s-1})x_s + w_s L - C_s - G_s, \qquad (52)$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u \Big[(1+r) B_s - B_{s+1}$$

$$- V_s (x_{s+1} - x_s) + d_s x_s + w_s L - G_s \Big]$$

$$V_s u'(C_s) = (V_{s+1} + d_{s+1}) \beta u'(C_{s+1}).$$

$$1 + r = \frac{d_{s+1} + V_{s+1}}{V_s}. \tag{53}$$

$$Q_{s+1} = B_{s+1} + V_s x_{s+1}.$$

$$d_s x_s + (V_s - V_{s-1}) x_s = r V_{s-1} x_s \tag{54}$$

$$Q_{s+1} - Q_s = r Q_s + w_s L - C_s - G_s.$$

$$Q_{t+1} = (1+r) B_t + d_t x_t + V_t x_t + w_t L - C_t - G_t.$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t$$

$$+ d_t x_t + V_t x_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s L - G_s). \qquad (55)$$

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} Q_{t+T+1} = 0,$$

$$V_t = \frac{d_{t+1}}{1+r} + \frac{V_{t+1}}{1+r}.$$

$$V_t = \frac{d_{t+1}}{1+r} + \frac{d_{t+2}}{(1+r)^2} + \frac{V_{t+2}}{(1+r)^2}.$$

$$V_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} d_s \tag{56}$$

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T} V_{t+T} = 0, \tag{57}$$

$$V_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \big[A_s F(K_s, L_s)$$

$$- w_s L_s - -(K_{s+1} - K_s)\big]. \tag{58}$$

$$d_t + V_t$$

$$= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_s F(K_s, L_s) - w_s L_s - (K_{s+1} - K_s)\right]$$

$$A_s F_K(K_s, L_s) = r$$

$$A_s F_L(K_s, L_s) = w_s.$$

$$x_s = 1, \qquad L_s = L$$

$$V_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[rK_s - (K_{s+1} - K_s)\right]$$

$$= \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[(1+r)K_s - K_{s+1}\right] = K_{t+1}$$

(59)

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = \left[(1+r)B_t + (1+r)K_t\right]$$

$$+ \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (w_s L - G_s)$$

$$= (1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (w_s L - G_s) \qquad (60)$$

$$C_t = rQ_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (w_s L - G_s). \qquad (61)$$

$$CA_t = \left[w_t L_t - \left(\widetilde{w_t L_t}\right)\right] - (G_t - \tilde{G}_t) - I_t,$$

$$S_t = \left[w_t L_t - \left(\widetilde{w_t L_t} \right) \right] - (G_t - \tilde{G}_t),$$

$$A_s F(K_s, L_s) - w_s L_s - (K_{s+1} - K_s).$$

$$d_t + V_t =$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[A_s F(K_s, L_s) - \frac{\chi}{2} (I_s^2 / K_s) - w_s L_s - I_s \right] \quad (62)$$

$$K_{s+1} - K_s = I_s.$$

$$\mathcal{L}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left\{ A_s F(K_s, L_s) - \frac{\chi}{2}(I_s^2/K_s) \right.$$

$$\left. - \, w_s L_s - I_s - q_s \left(K_{s+1} - K_s - I_s\right) \right\}.$$

$$-\frac{\chi I_s}{K_s} - 1 + q_s = 0.$$

$$I_s = \frac{q_s - 1}{\chi} K_s. \tag{63}$$

$$-q_s + \frac{A_{s+1} F_K(K_{s+1}, L_{s+1}) + \frac{\chi}{2}(I_{s+1}/K_{s+1})^2 + q_{s+1}}{1+r} = 0 \tag{64}$$

$$q_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t}$$

$$\left[A_{s+1}F_K(K_{s+1}, L_{s+1}) + \frac{\chi}{2}(I_{s+1}/K_{s+1})^2\right] \qquad (65)$$

$$K_{t+1} - K_t = \left(\frac{q_t - 1}{\chi}\right) K_t \qquad (66)$$

$$q_{t+1} - q_t = r q_t$$

$$- AF_K\left[K_t\left(1 + \frac{q_t - 1}{\chi}\right), L\right] - \frac{1}{2\chi}\left(q_{t+1} - 1\right)^2 \qquad (67)$$

$$K_{t+1} - K_t = \frac{\bar{K}}{\chi}(q_t - 1), \tag{68}$$

$$q_{t+1} - q_t = \left[r - \frac{A\bar{K}F_{KK}(\bar{K}, L)}{\chi} \right](q_t - 1)$$
$$- AF_{KK}(\bar{K}, L)(K_t - \bar{K}). \tag{69}$$

$$0 = \left[r - \frac{A\bar{K}F_{KK}(\bar{K}, L)}{\chi} \right](q - 1) - AF_{KK}(\bar{K}, L)(K - \bar{K}),$$

$$\left.\frac{\mathrm{d}q}{\mathrm{d}K}\right|_{\Delta q=0} = \frac{AF_{KK}(\bar{K}, L)}{r - A\bar{K}F_{KK}(\bar{K}, L)/\chi} < 0$$

$$q_t K_{t+1}$$

$$= \frac{A_{t+1} F_K(K_{t+1}, L_{t+1}) K_{t+1} + (\chi/2)(I_{t+1}^2/K_{t+1}) - q_{t+1} I_{t+1} + q_{t+1} K_{t+2}}{1+r}.$$

$$q_t K_{t+1}$$

$$= \frac{A_{t+1} F_K(K_{t+1}, L_{t+1}) K_{t+1} - (\chi/2)(I_{t+1}^2/K_{t+1}) - I_{t+1} + q_{t+1} K_{t+2}}{1+r}.$$

$$q_t K_{t+1}$$

$$= \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_s F(K_s, L_s) - \frac{\chi}{2}(I_s^2/K_s) - w_s L_s - I_s\right] = V_t; \quad (70)$$

$$r = \frac{A_{t+1}F_K(K_{t+1}, L_{t+1}) - (\chi/2)(I_{t+1}/K_{t+1})^2}{q_t}$$

$$\frac{-(I_{t+1}/K_{t+1}) + q_{t+1}(K_{t+2}/K_{t+1}) - q_t}{q_t}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = \left[(1+r)B_t + (1+r)q_{t-1}K_t\right]$$

$$+ \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s L_s - G_s) \qquad (71)$$

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t),$$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, \bar{L} - L_s),$$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)Q_t$$

$$+ \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (w_s L_s - G_s)$$

$$u_C(C_s, \bar{L} - L_s) = (1+r)\beta u_C(C_{s+1}, \bar{L} - L_{s+1}). \quad (72)$$

$$u_{\bar{L}-L}(C_s, \bar{L} - L_s) = u_C(C_s, \bar{L} - L_s)w_s. \quad (73)$$

$$u(C, \bar{L} - L) = \frac{1}{1 - 1/\sigma}\left[C^{\gamma}(\bar{L} - L)^{1-\gamma}\right]^{1-1/\sigma}.$$

$$\bar{L} - L = \frac{1 - \gamma}{\gamma w}C.$$

$$C_{s+1} = \left(\frac{w_s}{w_{s+1}}\right)^{(1-\gamma)(\sigma-1)} (1 + r)^{\sigma}\beta^{\sigma}C_s.$$

$$Y = AF(K) = AK^{\alpha}$$

$$A_{s+1} = (1+g)^{1-\alpha} A_s$$

$$I_s = K_{s+1} - K_s = \left(\frac{\alpha A_s}{r}\right)^{1/(1-\alpha)} g.$$

$$Y = AK^{\alpha} = A\left[\left(\frac{\alpha A}{r}\right)^{1/(1-\alpha)}\right]^{\alpha}$$

$$= A^{1/(1-\alpha)}\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}$$

$$I = \left(\frac{\alpha A}{r}\right)^{1/(1-\alpha)} g$$

$$= \frac{\alpha}{r}\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)} A^{1/(1-\alpha)} g = \left(\frac{\alpha g}{r}\right) Y$$

$$Y_s - I_s - G_s = (1+g)^{s-t}\left(1 - \frac{\alpha g}{r} - \varsigma\right) Y_t.$$

$$CA_t = Y_t - I_t - G_t - \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - I_s - G_s) - \frac{\vartheta}{1+r} W_t$$

$$= -\vartheta B_t - \frac{g+\vartheta}{r-g} \left(1 - \frac{\alpha g}{r} - \varsigma\right) Y_t. \tag{74}$$

$$\frac{B_{s+1}}{Y_{s+1}} = \left[\frac{(1+r)^{\sigma} \beta^{\sigma}}{1+g}\right] \frac{B_s}{Y_s}$$

$$- \frac{1+g-(1+r)^{\sigma}\beta^{\sigma}}{(1+g)(r-g)} \left(1 - \frac{\alpha g}{r} - \varsigma\right) \tag{75}$$

$$\overline{B/Y} = -\frac{1-\varsigma-(\alpha g/r)}{r-g}. \tag{76}$$

$$\frac{1}{(1+r)Y_t}\sum_{s=t}^{\infty}\left(\frac{1+g}{1+r}\right)^{s-t}(Y_t - G_t - I_t)$$

$$= \frac{Y_t - G_t - I_t}{(r-g)Y_t}$$

$$= \frac{1 - \varsigma - (\alpha g/r)}{r-g} = -\overline{B/Y}.$$

$$\frac{C}{Y} = \frac{r+\vartheta}{1+r}\left[(1+r)\frac{B}{Y} + \frac{1+r}{(r-g)}\left(\frac{Y-G-I}{Y}\right)\right]$$

$$= (r+\vartheta)\left[\frac{B}{Y} + \frac{Y-G-I}{(r-g)Y}\right]$$

$$C = Y - G - I = \frac{r + \vartheta}{1 + r}\left[\frac{1 + r}{r - g^*}(Y - G - I)\right]$$

$$1 + r = \frac{(1 + g^*)^{1/\sigma}}{\beta}. \tag{77}$$

$$\frac{B_{s+1}}{Y_{s+1}} - \frac{\bar{B}}{Y} = \left(\frac{1 + g^*}{1 + g}\right)\left(\frac{B_s}{Y_s} - \frac{\bar{B}}{Y}\right).$$

$$1 - \frac{1 + g^*}{1 + g} = \frac{g - g^*}{1 + g}$$

$$V_t u'(C_t) = \beta(d_{t+1} + V_{t+1})u'(C_{t+1}).$$

$$V_t u'(C_t) = \beta d_{t+1} u'(C_{t+1}) + \beta V_{t+1} u'(C_{t+1})$$

$$= \beta d_{t+1} u'(C_{t+1}) + \beta^2 d_{t+2} u'(C_{t+2})$$

$$+ \beta^2 V_{t+2} u'(C_{t+2}),$$

$$V_t u'(C_t) = \sum_{s=t+1}^{\infty} \beta^{s-t} u'(C_s) d_s$$

$$+ \lim_{T \to \infty} \beta^T u'(C_{t+T}) V_{t+T}. \tag{78}$$

$$\sum_{s=t+1}^{\infty} \beta^{s-t} u'(C_s) d_s.$$

$$V_t = \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(C_s)}{u'(C_t)} d_s = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} d_s,$$

$$\lim_{T\to\infty} \beta^T u'(C_{t+T}) V_{t+T} = 0.$$

$$\lim_{T\to\infty} (1+r)^{-T} V_{t+T} = 0.$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s+\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T}\left(B_{t+T+1}+V_{t+T}x_{t+T+1}\right)$$

$$=(1+r)B_t+d_t x_t+V_t x_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s L-G_s).$$

$$q_t=\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[A_{s+1}F_K(K_{s+1},L_{s+1})+\frac{\chi}{2}(I_{s+1}/K_{s+1})^2\right]$$

$$+\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T}q_{t+T}.$$

$$q_t>\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[A_{s+1}F_K(K_{s+1},L_{s+1})+\frac{\chi}{2}(I_{s+1}/K_{s+1})^2\right]$$

Table 2.1 Real External Debt Burdens of Selected Countries, 1970–91 (percent of GDP per year)

Country	**1970**	**1983**	**1991**
Argentina	0.5	2.9	3.9
Australia	1.7	1.3	2.4
Brazil	0.0	1.3	0.8
Canada	1.2	1.6	1.6
Chile	1.7	1.5	3.1
Hungary	0.0	2.3	3.8
Mexico	0.1	3.1	1.5
Nigeria	0.1	1.1	4.8
Thailand	0.0	0.0	0.2

Source: Authors' calculations based on data from World Bank, *World Development Report,* various issues.

Table 2.2 Pooled Time-Series Regressions for the G-7 Countries, 1961–90

Dependent Variable	a_1	a_2	a_3
ΔI	0.35	0.53	–0.10
	(0.03)	(0.06)	(0.04)
	b_1	b_2	b_3
ΔCA	–0.17	0.01	0.04
	(0.03)	(0.02)	(0.03)

Standard errors in parentheses. Regressions include a time trend.
Source: Glick and Rogoff (1995).