1 Intertemporal Trade and the Current Account Balance

$$U_1^i = u(c_1^i) + \beta u(c_2^i), \qquad 0 < \beta < 1. \tag{1}$$

$$c_1^i + \frac{c_2^i}{1+r} = y_1^i + \frac{y_2^i}{1+r}. \tag{2}$$

$$\max_{c_1^i} u(c_1^i) + \beta u[(1+r)(y_1^i - c_1^i) + y_2^i].$$

$$u'(c_1^i) = (1+r)\beta u'(c_2^i), \tag{3}$$

$$\frac{\beta u'(c_2^i)}{u'(c_1^i)} = \frac{1}{1+r}. \tag{4}$$

$$\bar{c}^i = \frac{[(1+r)y_1^i + y_2^i]}{2+r}. \tag{5}$$

$$CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t, \tag{6}$$

$$CA_2 = Y_2 + rB_2 - C_2 = Y_2 + r(Y_1 - C_1) - C_2$$

$$= -(Y_1 - C_1) = -B_2 = -CA_1,$$

$$C_2 = Y_2 - (1 + r)(C_1 - Y_1).$$

$$\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1 + r^{\mathrm{A}}}. \tag{7}$$

$$\frac{u'(Y_2)}{u'(Y_1)} = \frac{1 + r}{1 + r^{\mathrm{A}}},$$

$$C_1 + \frac{C_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}. \tag{8}$$

$$CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t - G_t.$$

$$\bar{C} = \frac{[(1+r)(\bar{Y} - G_1) + \bar{Y}]}{2+r} = \bar{Y} - \frac{(1+r)G_1}{2+r}.$$

$$CA_1 = \bar{Y} - \bar{C} - G_1 = -\frac{G_1}{2+r} < 0.$$

$$U_1 = \sum_{t=1}^{T} \beta^{t-1} u(C_t). \tag{9}$$

$$Y = F(K). \tag{10}$$

$$K_{t+1} = K_t + I_t. \tag{11}$$

$$B_{t+1} + K_{t+1} - (B_t + K_t) = Y_t + r_t B_t - C_t - G_t.$$

$$CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t - G_t - I_t. \tag{12}$$

$$S_t \equiv Y_t + r_t B_t - C_t - G_t. \tag{13}$$

$$CA_t = S_t - I_t. \tag{14}$$

$$B_2 = Y_1 - C_1 - G_1 - I_1$$

$$-B_2 = Y_2 + rB_2 - C_2 - G_2 - I_2$$

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}. \quad (15)$$

$$I_2 = K_3 - K_2 = 0 - K_2 = -K_2.$$

$$\max_{C_1, I_1} u(C_1) + \beta u \left\{(1+r)\left[F(K_1) - C_1 - G_1 - I_1\right] + F(I_1 + K_1) - G_2 + I_1 + K_1\right\}. \quad (16)$$

$$F'(K_2) = r, \tag{17}$$

$$C_2 = F\left[K_1 + F(K_1) - C_1\right] + K_1 + F(K_1) - C_1. \tag{18}$$

$$\frac{\mathrm{d}C_2}{\mathrm{d}C_1} = -[1 + F'(K_2)].$$

$$Y_t + Y_t^* = C_t + C_t^*.$$

$$S_t + S_t^* = 0.$$

$$S_1 + S_1^* = 0. \tag{19}$$

$$r^{\mathrm{A}} < r < r^{\mathrm{A}*}.$$

$$\mathrm{d}\log(1+r) = \frac{u''(C_1)}{u'(C_1)}\mathrm{d}C_1 - \frac{u''(C_2)}{u'(C_2)}\mathrm{d}C_2$$

$$= \frac{C_1 u''(C_1)}{u'(C_1)}\mathrm{d}\log C_1 - \frac{C_2 u''(C_2)}{u'(C_2)}\mathrm{d}\log C_2. \tag{20}$$

$$\sigma(C) = -\frac{u'(C)}{Cu''(C)}. \tag{21}$$

$$\mathrm{d}\log\left(\frac{C_2}{C_1}\right) = \sigma\,\mathrm{d}\log(1+r).$$

$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \qquad \sigma > 0. \tag{22}$$

$$u'(C_1) = (1+r)\beta u'\left[(1+r)(Y_1 - C_1) + Y_2\right].$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}r} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \tag{23}$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}r} = \frac{(Y_1 - C_1) - \sigma C_2/(1+r)}{1 + r + (C_2/C_1)}. \tag{24}$$

$$C_2 = (1+r)^{\sigma} \beta^{\sigma} C_1. \tag{25}$$

$$C_1 = \frac{1}{1 + (1+r)^{\sigma-1}\beta^{\sigma}} \left(Y_1 + \frac{Y_2}{1+r} \right). \tag{26}$$

$$Y = AF(K), \qquad Y^* = A^* F^*(K^*),$$

$$u'(C_1) = (1+r)\beta u' \Big\{ (1+r)[A_1 F(K_1) - C_1 - I_1] \\ + A_2 F(K_1 + I_1) + K_1 + I_1 \Big\}$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}r} = \\ \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\Big\{[A_1 F(K_1) - C_1 - I_1] + \left[A_2 F'(K_1 + I_1) - r\right] \frac{\partial I_1}{\partial r}\Big\}}{u''(C_1) + \beta(1+r)^2 u''(C_2)},$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}r} = \frac{(Y_1 - C_1 - I_1) - \sigma C_2/(1+r)}{1 + r + (C_2/C_1)},$$

$$Y_1 + Y_1^* = C_1 + C_1^* + I_1 + I_1^*$$

$$S_1 + S_1^* = I_1 + I_1^*.$$

$$CA_1 + CA_1^* = 0.$$

$$\left.\frac{\mathrm{d}I_1}{\mathrm{d}A_2}\right|_{r\ \text{constant}} = -\frac{F'(K_2)}{A_2 F''(K_2)} > 0.$$

$$I_1 + I_1^* = (A_2\alpha/r)^{1/(1-\alpha)} - K_1.$$

$$\mathrm{d}r|_{I+I^*\ \text{constant}} = r\frac{\mathrm{d}A_2}{A_2} = r\hat{A}_2,$$

$$S_1 + S_1^* = \frac{\beta}{1+\beta}A_1 F(K_1) + \frac{1}{1+\beta}\left[K_2 - K_1 - \frac{A_2 F(K_2) + K_2}{1+r}\right].$$

$$
\begin{aligned}
&\frac{-1}{1+\beta}\left[\frac{F(K_2)}{1+r}\mathrm{d}A_2 - \frac{A_2F(K_2)+K_2}{(1+r)^2}\mathrm{d}r\right] \\
&\qquad = \frac{-1}{1+\beta}\left[\frac{A_2F(K_2)}{1+r}\hat{A}_2 - \frac{A_2F(K_2)+K_2}{(1+r)^2}\mathrm{d}r\right] \\
&\qquad = \mathrm{d}(S_1+S_1^*) = 0.
\end{aligned}
$$

$$dr|_{S+S^* \text{ constant}}$$

$$= (1+r)\frac{A_2F(K_2)}{A_2F(K_2)+K_2}\hat{A}_2$$

$$= (1+r)\frac{A_2^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}}{A_2^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}+A_2^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}}\hat{A}_2$$

$$= \frac{1+r}{1+\frac{\alpha}{r}}\hat{A}_2 > r\hat{A}_2 = dr|_{I+I^* \text{ constant}} \cdot$$

$$R \equiv \frac{1}{1+r}$$

$$C_2^{\mathrm{H}}(R, U_1) = E_R\,(R, U_1)\,. \tag{27}$$

$$E(R, U_1) = Y_1 + RY_2.$$

$$E_R \mathrm{d}R + E_U \mathrm{d}U_1 = Y_2 \mathrm{d}R,$$

$$E_U \frac{\mathrm{d}U_1}{\mathrm{d}R} = Y_2 - C_2^{\mathrm{H}}(R, U_1) = Y_2 - C_2. \tag{28}$$

$$W_1 \equiv Y_1 + RY_2.$$

$$C_1(R, W_1) = \frac{W_1}{1 + \beta^\sigma R^{1-\sigma}} = \frac{Y_1 + RY_2}{1 + \beta^\sigma R^{1-\sigma}},$$

$$C_1 \left[R, E(R, U_1) \right] = C_1^{\mathrm{H}}(R, U_1). \tag{29}$$

$$\frac{\partial C_1(R, W_1)}{\partial R} = \frac{\partial C_1^{\mathrm{H}}(R, U_1)}{\partial R} - \frac{\partial C_1}{\partial W_1} C_2. \tag{30}$$

$$\frac{\mathrm{d}C_1(R, W_1)}{\mathrm{d}R} = \frac{\partial C_1(R, W_1)}{\partial R} + \frac{\partial C_1(R, W_1)}{\partial W_1}\frac{\mathrm{d}W_1}{\mathrm{d}R}$$

$$= \frac{\partial C_1^{\mathrm{H}}(R, U_1)}{\partial R} + \frac{\partial C_1(R, W_1)}{\partial W_1}(Y_2 - C_2). \tag{31}$$

$$C_1^{\mathrm{H}}(R, U_1) = \left[\frac{\left(1 - \frac{1}{\sigma}\right) U_1}{1 + \beta^{\sigma} R^{1-\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}R} = \sigma \frac{\beta^{\sigma} R^{-\sigma}}{1 + \beta^{\sigma} R^{1-\sigma}} C_1 + \frac{1}{1 + \beta^{\sigma} R^{1-\sigma}}(Y_2 - C_2).$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}R} = (\sigma - 1)\frac{\beta^\sigma R^{-\sigma}}{1+\beta^\sigma R^{1-\sigma}}C_1 + \frac{1}{1+\beta^\sigma R^{1-\sigma}}Y_2.$$

$$S_1^*(r) = Y_1^* - C_1^*(r) = \frac{\beta^*}{1+\beta^*}Y_1^*\frac{1}{(1+\beta^*)(1+r)}Y_2^*$$

$$1 + r = \frac{Y_2^*}{(1+\beta^*)(Y_1 - C_1) + \beta^* Y_1^*}.$$

$$C_2 = Y_2 + \frac{Y_2^*}{(1+\beta^*)(Y_1 - C_1) + \beta^* Y_1^*}(Y_1 - C_1). \quad (32)$$

$$\left.\frac{\mathrm{d}C_2}{\mathrm{d}C_1}\right|_{C_1=Y_1} = -\frac{Y_2^*}{\beta Y_1^*} = -(1 + r^{\mathrm{A}*})$$

$$Y = F(K, L). \tag{33}$$

$$Y = F(K, L) = F_K(K, L)K + F_L(K, L)L \tag{34}$$

$$F_K(K, L) = f'(k), \tag{35}$$

$$F_L(K, L) = f(k) - f'(k)k. \tag{36}$$

$$Y_2 = F(K_2, L_2),$$

$$C_1 = Y_1 - K_2,$$

$$C_2 = L_2 f(K_2/L_2) - w(L_2 - L^{\mathrm{H}}) + K_2.$$

$$\max_{K_2, L_2} u(Y_1 - K_2)$$

$$+ \beta u\left[L_2 f(K_2/L_2) - w(L_2 - L^{\mathrm{H}}) + K_2\right]$$

$$u'(C_1) = \beta[1 + f'(k_2)]u'(C_2), \tag{37}$$

$$w = f(k_2) - f'(k_2)k_2, \tag{38}$$

$$C_2 = F\left(Y_1 - C_1, L^{\mathrm{H}}\right) + Y_1 - C_1.$$

$$C_2 = [1 + r(w)](Y_1 - C_1) + wL^{\mathrm{H}},$$

$$Y_2 + K_2 = F(K_2, L_2) + K_2 = [1 + r(w)]K_2 + wL_2$$

$$= \left[1 + r(w) + \frac{w}{k(w)}\right](Y_1 - C_1).$$

$$S_1(r) + S_1^*(r) = I_1(r) + I_1^*(r).$$

$$S_1'(r) + S_1^{*\prime}(r) - I_1'(r) - I_1^{*\prime}(r) > 0. \tag{39}$$

$$IM_1 = \left(\frac{1}{1 + r}\right) EX_2,$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(\frac{1}{1+r}\right) IM_2^*(r) - IM_1(r)\right] > 0. \tag{40}$$

$$\zeta \equiv -\frac{(1+r)IM_1'(r)}{IM_1(r)},\ \zeta^* = \frac{(1+r)IM_2^{*\prime}(r)}{IM_2^*(r)}$$

$$\zeta + \zeta^* > 1. \tag{41}$$

Table 1.1 GNP versus GDP for Selected Countries, 1990 (dollars per capita)

Country	GDP	GNP	Percent Difference
Australia	17,327	17,000	−1.9
Brazil	2,753	2,680	−2.7
Canada	21,515	20,470	−4.9
Saudi Arabia	5,429	7,050	29.9
Singapore	11,533	11,160	−3.2
United Arab Emirates	17,669	19,860	12.4
United States	21,569	21,790	1.0

Source: World Bank, *World Development Report 1992*.

Table 1.2 Japan's Gross Saving and Investment During the Russo-Japanese War (fraction of GDP)

Year	**Saving/GDP**	**Investment/GDP**
1903	0.131	0.136
1904	0.074	0.120
1905	0.058	0.168
1906	0.153	0.164

Table 1.3 Industrial Country Net Immigration (average per year, as a percent of labor force)

Country	1870–1913	1914–49	1950–73	1974–87
Australia	0.96	0.74	2.06	1.14
Belgium	0.15	0.16	0.34	–0.02
Canada	1.08	0.15	1.35	0.50
France	0.11	–0.03	0.75	0.11
Germany	–0.48	–0.08	1.12	0.26
Italy	–0.64	–0.25	–0.41	0.17
Japan	n.a.	0.02	–0.01	–0.02
Netherlands	–0.18	–0.03	0.04	–0.02
Norway	–1.63	–0.33	0.00	0.30
Sweden	–0.92	0.06	0.38	0.28
Switzerland	0.07	–0.15	1.15	0.00
United Kingdom	–0.97	–0.24	–0.10	0.00
United States	1.38	0.35	0.47	0.51

Source: Maddison (1991) and OECD, *Labor Force Statistics*. The figures were calculated by dividing Maddison's average net immigration series by labor force membership as of 1890, 1929, 1960, and 1981.

Table 1.4 Gross Saving as a Percent of GDP (period average)

Country Group	**1973–80**	**1981–87**	**1988–94**
Industrial	23.5	20.9	20.5
Fuel exporting	42.0	20.2	19.7
Nonfuel developing	22.4	22.9	26.2

Source: International Monetary Fund, *World Economic Outlook*, October 1995.