Foundations of International Macroeconomics[1]

Workbook[2]

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Chapter 9 Solutions

1. (a) In the problem, the government unexpectedly freezes the money supply, which had previously been growing predictably at the proportional rate μ, at the *previous* period's level $\overline{\mathsf{m}}$, say. Since the economy was initially in a steady state, we have that $\mathsf{p}_0 = \overline{\mathsf{m}} + (1+\eta)\mu$, which, together with eq. (12) in Chapter 9, implies that

$$\mathsf{q}_0 = \mathsf{e}_0 - \overline{\mathsf{m}} - (1+\eta)\mu. \tag{1}$$

(Recall that $\bar{\mathsf{y}}$, i^*, $\mathsf{p}^* \equiv 0$. This problem differs from the one in section 9.2.5 in that here, there is a forecast error in the level of the date 0 money supply, whereas in the chapter, there is no surprise concerning the date 0 money supply.) Let us reproduce eq. (18) from Chapter 9 (which assumes a constant long-run real exchange rate):

$$\mathsf{e}_t - \mathsf{e}_t^{flex} = \frac{1-\phi\delta}{1+\psi\delta\eta}(\mathsf{q}_t - \overline{\mathsf{q}}).$$

For $t = 0$, we may use eq. (1) to eliminate q_0 from the preceding equation:

$$\mathsf{e}_0 - \mathsf{e}_0^{flex} = \frac{1-\phi\delta}{1+\psi\delta\eta}\left[\mathsf{e}_0 - \overline{\mathsf{m}} - (1+\eta)\mu - \overline{\mathsf{q}}\right].$$

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Observe that e_0^{flex}, the post-disturbance flexible-price exchange rate, simply equals $\overline{\mathsf{q}}+\overline{\mathsf{m}}$ (because the money supply has been frozen at its date $t=-1$ level). The preceding equation therefore becomes

$$\mathsf{e}_0=\overline{\mathsf{q}}+\overline{\mathsf{m}}-\left(\frac{1-\phi\delta}{\psi\delta\eta+\phi\delta}\right)(1+\eta)\mu=\mathsf{e}_0^{flex}-\left(\frac{1-\phi\delta}{\psi\delta\eta+\phi\delta}\right)(1+\eta)\mu.$$

The effect on the real exchange rate (relative to the economy's initial path) is the difference $\mathsf{e}_0-\mathsf{p}_0-\overline{\mathsf{q}}=\mathsf{e}_0-[\overline{\mathsf{m}}+(1+\eta)\mu]-\overline{\mathsf{q}}$, which, from the last displayed equation equals

$$-\left(\frac{1-\phi\delta}{\psi\delta\eta+\phi\delta}\right)(1+\eta)\mu-(1+\eta)\mu=-\left(\frac{1+\psi\delta\eta}{\psi\delta\eta+\phi\delta}\right)(1+\eta)\mu.$$

Thus, there is an initial real appreciation on date 0, after which the real exchange rate converges back to its long-run level $\overline{\mathsf{q}}$ according to eq. (13) in Chapter 9. [The real appreciation is proportional to the total monetary shock, equal to the money-supply level shock, $-\mu$, plus the money-supply growth rate effect, $-\eta\mu$, with the same proportionality factor as in eq. (17), Chapter 9.] From the aggregate demand equation in the chapter, eq. (3), we see that output falls initially by a percentage equal to δ times the percentage real appreciation, and then gradually rises back to its full-employment level. Footnote 17 in the chapter implies that the initial Home-less-Foreign real interest differential is the expected change in q after the shock hits. Equation (13) in the chapter shows that

$$\begin{aligned}\mathsf{q}_1-\mathsf{q}_0 &= \mathsf{q}_1-\overline{\mathsf{q}}-(\mathsf{q}_0-\overline{\mathsf{q}})\\ &= -\psi\delta\,(\mathsf{q}_0-\overline{\mathsf{q}})\\ &= \psi\delta\left(\frac{1+\psi\delta\eta}{\psi\delta\eta+\phi\delta}\right)(1+\eta)\mu.\end{aligned}$$

Thus Home's relative real interest rate rises, a result that also can be derived from eq. (24) in Chapter 9.

(b) To avoid the short-run disruptive effects, the monetary authority could unexpectedly increase the money supply by $(1+\eta)\mu$ percent on date 0, while reducing money growth between periods 1 and 0, 2 and 1, etc., to zero. The inflation rates $\mathsf{p}_1 - \mathsf{p}_0$, $\mathsf{p}_2 - \mathsf{p}_1$, etc., would still all be zero, as in the strategy described in part a; $\mathsf{p}_0 - \mathsf{p}_{-1}$ would still equal μ as in part a. With the date 0 money supply equal to $\overline{\mathsf{m}}+(1+\eta)\mu$ rather than $\overline{\mathsf{m}}$, however, $\mathsf{p}_0 = \overline{\mathsf{m}}+(1+\eta)\mu$ would equal the new long-run price level. The difficulty with such a plan is its credibility—how does one convince markets in period 0 that the inflation-stabilization plan is serious without slowing money-supply growth in period 0 itself? Achieving such credibility is critical, for if the market does not believe future money growth will be curbed, the expected inflation term in the Phillips curve will remain active and the liquidity squeeze and recession will occur in period 1 (when the money supply finally is frozen) rather than in period 0. Interestingly, an equivalent way to proceed would be simply to suddenly peg the exchange rate at the level $\overline{\mathsf{q}} + \mathsf{p}_0 = \overline{\mathsf{q}} + \overline{\mathsf{m}} + (1+\eta)\mu$, while simultaneously announcing that the exchange rate is henceforth irrevocably fixed. If credible, such an announcement will bring the Home nominal interest rate down to the Foreign level as capital inflows swell the economy's money supply and prevent a liquidity squeeze. The credibility problems raised by this exchange-rate based inflation stabilization approach are the subject of a large literature (since the approach has been tried so often, frequently with unfortunate final results). For a survey see Carlos A. Végh, "Stopping high inflation: An analytical overview," *International Monetary Fund Staff Papers* 39 (September 1992): 626-95.

2. To do this question, we shall first find the generalized saddle path relation between the nominal and real exchange rates that holds when the long-run real exchange rate can vary exogenously (for example, due to shifts in foreign demand for domestic exports). Let us begin by reformulating eq. (3) in Chapter 9 as

$$\mathsf{y}_t^d = \overline{\mathsf{y}} + \delta(\mathsf{e}_t + \mathsf{p}^* - \mathsf{p}_t - \overline{\mathsf{q}}_t), \tag{2}$$

so that we allow for a time-varying $\overline{\mathsf{q}}$. Following the logic that led to eq. (6)

in the chapter, we now obtain instead:

$$\mathsf{p}_{t+1} - \mathsf{p}_t = \psi(\mathsf{y}_t^d - \overline{\mathsf{y}}) + \mathsf{e}_{t+1} - \mathsf{e}_t - (\overline{\mathsf{q}}_{t+1} - \overline{\mathsf{q}}_t).$$

Using eq. (2) above to eliminate $\mathsf{y}_t^d - \overline{\mathsf{y}}$ here, and using the definition of the real exchange rate, we find that the real exchange rate follows the difference equation

$$\mathsf{q}_{t+1} - \mathsf{q}_t = -\psi\delta(\mathsf{q}_t - \overline{\mathsf{q}}_t) + \overline{\mathsf{q}}_{t+1} - \overline{\mathsf{q}}_t.$$

Since this equation can be expressed entirely in terms of the deviations $\mathsf{q} - \overline{\mathsf{q}}$, its solution is the same as eq. (13) in the chapter, but with a variable long-run real exchange rate:

$$\mathsf{q}_s - \overline{\mathsf{q}}_s = (1 - \psi\delta)^{s-t}(\mathsf{q}_t - \overline{\mathsf{q}}_t). \tag{3}$$

Following the steps leading to eq. (9) in the chapter and assuming that the money supply is constant at $\overline{\mathsf{m}}$ yields

$$\mathsf{e}_{t+1} - \mathsf{e}_t = \frac{\mathsf{e}_t}{\eta} - \frac{(1-\phi\delta)\mathsf{q}_t}{\eta} - \left(\frac{\phi\delta\overline{\mathsf{q}}_t + \overline{\mathsf{m}}}{\eta}\right).$$

Define $\overline{\mathsf{e}}_t$ as the flexible-price equilibrium exchange rate. (This variable is the same as the flexible-price exchange rate as defined on p. 618, e_t^{flex} but it will prove easier on the eyes to use the overbar notation instead.) Since monetary equilibrium condition (2) from the chapter also holds in a hypothetical flexible price equilibrium, it is also true that the last equation holds with overbars,

$$\overline{\mathsf{e}}_{t+1} - \overline{\mathsf{e}}_t = \frac{\overline{\mathsf{e}}_t}{\eta} - \frac{(1-\phi\delta)\overline{\mathsf{q}}_t}{\eta} - \left(\frac{\phi\delta\overline{\mathsf{q}}_t + \overline{\mathsf{m}}}{\eta}\right). \tag{4}$$

Now subtract this equation from the one preceding it to obtain the analog of the equation preceding eq. (14) in Chapter 9:

$$\mathsf{e}_t - \overline{\mathsf{e}}_t = \frac{\eta}{1+\eta}\left(\mathsf{e}_{t+1} - \overline{\mathsf{e}}_{t+1}\right) + \frac{1-\phi\delta}{1+\eta}\left(\mathsf{q}_t - \overline{\mathsf{q}}_t\right).$$

Now we can calculate the economy's saddle path as in the book, utilizing eq. (3) from above rather than eq. (13) in Chapter 9. The resulting generalized saddle path relation [after solving forward, excluding bubbles, and using (3) above] is

$$\mathsf{e}_t - \overline{\mathsf{e}}_t = \frac{1-\phi\delta}{1+\psi\delta\eta}\left(\mathsf{q}_t - \overline{\mathsf{q}}_t\right), \tag{5}$$

which generalizes eq. (16) in Chapter 9. We can finally proceed to answer the original question concerning an anticipated future rise in the equilibrium real exchange rate. Solve equation (4) forward (assuming no bubbles); the result is

$$\overline{\mathsf{e}}_t = \overline{\mathsf{m}} + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}\overline{\mathsf{q}}_s.$$

The exercise assumes that the economy starts out in a steady-state equilibrium on date $t = 0$, when it is announced that the equilibrium real exchange rate will rise from $\overline{\mathsf{q}}$ to $\overline{\mathsf{q}}'$ on date T in the future. Thus, the preceding equation implies that the flex-price nominal exchange rate jumps up on date $t = 0$ from $\overline{\mathsf{m}} + \overline{\mathsf{q}}$ to

$$\overline{\mathsf{e}}_0 = \overline{\mathsf{m}} + \left[1 - \left(\frac{\eta}{1+\eta}\right)^T\right]\overline{\mathsf{q}} + \left(\frac{\eta}{1+\eta}\right)^T\overline{\mathsf{q}}' > \overline{\mathsf{m}} + \overline{\mathsf{q}}.$$

Using eq. (5) above and the fact that $\mathsf{p}_0 = \overline{\mathsf{m}}$, we derive

$$\mathsf{e}_0 - (\overline{\mathsf{m}} + \overline{\mathsf{q}}) = \left(\frac{1+\psi\delta\eta}{\psi\delta\eta + \phi\delta}\right)\left[\overline{\mathsf{e}}_0 - (\overline{\mathsf{m}} + \overline{\mathsf{q}})\right] = \mathsf{q}_0 - \overline{\mathsf{q}} > 0.$$

We have now assembled enough information to characterize the economy's path after the initial real and nominal currency depreciations on date 0 (which induce higher aggregate demand and a rising price level). The economy's entire path is illustrated in figures 9.1 and 9.2, where the over- and undershooting cases are shown separately. According to eq. (3) above, the domestic currency *appreciates* in real terms between dates 0 and T as the initial gain in competitiveness erodes. Notwithstanding this effect, eq. (5)

above implies that the currency continues to depreciate in nominal terms between dates 0 and T. During that time interval, according to eq. (5) above,

$$\mathsf{e}_t = \overline{\mathsf{m}} + \left[1 - \left(\frac{\eta}{1+\eta}\right)^{T-t}\right]\overline{\mathsf{q}} + \left(\frac{\eta}{1+\eta}\right)^{T-t}\overline{\mathsf{q}}' + \frac{1-\phi\delta}{1+\psi\delta\eta}(1-\psi\delta)^t\left(\mathsf{q}_0 - \overline{\mathsf{q}}\right),$$

an expression that rises as $t \to T$. (We know the domestic currency must be depreciating at $t = 0$ because the impact rise in output, given the predetermined price level, implies a higher domestic nominal interest rate and therefore an expected domestic currency depreciation. However, if the preceding displayed equation satisfies $\mathrm{d}e_t/\mathrm{d}t > 0$ when evaluated at $t = 0$, it must also satisfy that inequality for $0 < t < T$, because, as you can easily see, $\mathrm{d}^2 e_t/\mathrm{d}t^2 > 0$ until $t = T$. Thus, the currency's depreciation actually accelerates prior to date T.) What happens on date T itself? The anticipated rise from $\overline{\mathsf{q}}$ to $\overline{\mathsf{q}}'$ occurs, the price level jumps downward (because the exogenous change in demand was fully anticipated), and, according to eq. (3) above and the unnumbered equation preceding it, the real exchange rate jumps upward by the amount

$$\mathsf{q}_T - \mathsf{q}_{T-1} = (\overline{\mathsf{q}}' - \overline{\mathsf{q}}) - \psi\delta(1-\psi\delta)^{T-1}(\mathsf{q}_0 - \overline{\mathsf{q}}) > 0.$$

By eqs. (3) and (5), the nominal exchange rate's movement is

$$\mathsf{e}_T - \mathsf{e}_{T-1} = \frac{1}{1+\eta}(\overline{\mathsf{q}}' - \overline{\mathsf{q}}) - \left(\frac{1-\phi\delta}{1+\psi\delta\eta}\right)\psi\delta(1-\psi\delta)^{T-1}(\mathsf{q}_0 - \overline{\mathsf{q}}).$$

Starting on date T, eq. (5) gives the nominal exchange rate's path as

$$\mathsf{e}_t - (\overline{\mathsf{m}} + \overline{\mathsf{q}}') = \frac{1-\phi\delta}{1+\psi\delta\eta}(1-\psi\delta)^{t-T}\left(\mathsf{q}_T - \overline{\mathsf{q}}'\right). \tag{6}$$

According to eq. (3) above,

$$\mathsf{q}_T - \overline{\mathsf{q}}' = (1-\psi\delta)^T(\mathsf{q}_0 - \overline{\mathsf{q}}) > 0.$$

Equation (6) above therefore implies that for $1-\phi\delta>0$ (the overshooting case), the date T nominal exchange rate is above its new long-run level $\overline{\mathsf{m}}+\overline{\mathsf{q}}'$, but that the gap is reduced over time (implying nominal currency appreciation from date T on). For $1-\phi\delta<0$ (the undershooting case) e_T stands below its new long-run level $\overline{\mathsf{m}}+\overline{\mathsf{q}}'$, but subsequently the currency depreciates in nominal terms toward its steady-state value. In both the over- and undershooting cases, the real exchange rate is above $\overline{\mathsf{q}}'$ on date T, and therefore the currency appreciates in real terms afterward (that is, q falls after date T, asymptotically approaching $\overline{\mathsf{q}}'$).

3. (a) Since the only factors buffeting the economy are the exogenous shocks ϵ and ν, since these are mean-zero i.i.d. shocks, and since the model contains no intrinsic persistence mechanisms, the economy is always expected next period to be at the equilibrium associated with the realizations $\epsilon=\nu=\mathsf{p}_t-\mathrm{E}_{t-1}\mathsf{p}_t=0$. It is easy to show that this equilibrium is characterized by $\mathsf{e}=\mathsf{p}=\overline{\mathsf{m}}$ and $\mathsf{y}=\mathsf{i}=0$. To solve for the equilibrium more generally, note that because we therefore have that $\mathrm{E}_{t-1}\mathsf{p}_t=\overline{\mathsf{m}}$, the equality $\mathsf{y}^s=\mathsf{y}^d$ can be written

$$\theta(\mathsf{p}_t-\overline{\mathsf{m}})=\delta(\mathsf{e}_t-\mathsf{p}_t)+\epsilon_t.$$

Since $\mathrm{E}_t\mathsf{e}_{t+1}=\overline{\mathsf{m}}$, the interest parity condition is $\mathsf{i}_{t+1}=\overline{\mathsf{m}}-\mathsf{e}_t$; substituting this into the monetary equilibrium condition gives

$$(1+\eta+\phi\theta)\overline{\mathsf{m}}-(1+\phi\theta)\mathsf{p}_t=\eta\mathsf{e}_t+\nu_t.$$

Combining the preceding two equations to solve for e and p, and then using the aggregate supply schedule to calculate equilibrium output y, we find that

$$\mathsf{e}_t=\overline{\mathsf{m}}-\frac{(1+\phi\theta)\epsilon_t+(\theta+\delta)\nu_t}{\delta(1+\phi\theta)+\eta(\theta+\delta)},$$

$$\mathsf{p}_t=\overline{\mathsf{m}}+\frac{\eta\epsilon_t-\delta\nu_t}{\delta(1+\phi\theta)+\eta(\theta+\delta)},$$

$$\mathsf{y}_t=\frac{\theta(\eta\epsilon_t-\delta\nu_t)}{\delta(1+\phi\theta)+\eta(\theta+\delta)}.$$

Using the last of these solutions, we see that $\mathrm{E}_{t-1}\{\mathsf{y}_t\}^2$—which equals the variance of output, since $\mathrm{E}_{t-1}\mathsf{y}_t = 0$—is given by

$$\mathrm{E}_{t-1}\{\mathsf{y}_t\}^2 = \frac{\theta^2\left(\eta^2\sigma_\epsilon^2 + \delta^2\sigma_\nu^2\right)}{\left[\delta(1+\phi\theta)+\eta(\theta+\delta)\right]^2}.$$

(Recall the assumption that ϵ and ν are distributed independently of each other, making their covariance zero.)

(b) When the exchange rate is fixed at $\bar{\mathsf{e}}$, $\mathrm{E}_{t-1}\mathsf{p}_t = \bar{\mathsf{e}} = \bar{\mathsf{m}}$. The equality $\mathsf{y}^s = \mathsf{y}^d$ therefore can be written as

$$\theta(\mathsf{p}_t - \bar{\mathsf{e}}) = \delta(\bar{\mathsf{e}} - \mathsf{p}_t) + \epsilon_t,$$

implying that

$$\mathsf{p}_t = \bar{\mathsf{e}} + \frac{\epsilon_t}{\theta+\delta},\ \mathsf{y}_t = \frac{\theta\epsilon_t}{\theta+\delta},\ \mathrm{E}_{t-1}\{\mathsf{y}_t\}^2 = \frac{\theta^2\sigma_\epsilon^2}{(\theta+\delta)^2}.$$

(c) The limiting behavior $\sigma_\varepsilon^2/\sigma_v^2 \to 0$ means that the variance of real (aggregate-demand) shocks is becoming negligible relative to that of monetary shocks. But the fixed exchange rate policy in part b *eliminates* the effect of the monetary shock ν on output! Any changes in money demand are automatically accommodated through foreign-exchange intervention, with no side-effects on output. A fixed exchange rate thus is bound to be better when money-market shocks dominate. More formally, combining the bottom line results in parts a and b, we see that the ratio

$$\frac{\mathrm{E}_{t-1}\{\mathsf{y}_t\}^2|_{\text{FLEX}}}{\mathrm{E}_{t-1}\{\mathsf{y}_t\}^2|_{\text{FIX}}} = \frac{(\theta+\delta)^2}{\left[\delta(1+\phi\theta)+\eta(\theta+\delta)\right]^2} \times \left[\eta^2 + \delta^2\left(\sigma_\nu^2/\sigma_\epsilon^2\right)\right].$$

As $\sigma_\varepsilon^2/\sigma_v^2 \to 0$, $\sigma_\nu^2/\sigma_\epsilon^2 \to \infty$, and thus the variability of output under a constant money supply rule becomes arbitrarily large compared to that under a constant exchange rate rule.

(d) [There are two typos in the statement of this part of the exercise. The monetary policy rule should be as given below, rather than $\mathsf{m}_t - \overline{\mathsf{m}} = \Phi(\mathsf{e}_t - \overline{\mathsf{e}})$ as in the book. Also, the optimal value of Φ lies between $-\eta$ and ∞, not 0 and ∞.] To analyze a monetary policy rule of the form

$$\mathsf{m}_t - \overline{\mathsf{m}} = \Phi(\overline{\mathsf{e}} - \mathsf{e}_t),$$

we use the following trick. We have already noted that in this particular model (with i.i.d. shocks), we may write the interest parity condition as $\mathsf{i}_{t+1} = \overline{\mathsf{e}} - \mathsf{e}_t$, so that the money-market equilibrium condition is

$$\mathsf{m}_t - \mathsf{p}_t = -\eta(\overline{\mathsf{e}} - \mathsf{e}_t) + \phi \mathsf{y}_t + \nu_t.$$

However, if we now substitute the policy rule for m_t into this money-market equilibrium equation, we can write the latter as

$$\overline{\mathsf{m}} - \mathsf{p}_t = -(\eta + \Phi)(\overline{\mathsf{e}} - \mathsf{e}_t) + \phi \mathsf{y}_t + \nu_t.$$

In short, the intervention policy rule leaves us with a model isomorphic to a model with a fixed money supply in which the interest semi-elasticity of money demand is

$$\tilde{\eta} \equiv \eta + \Phi.$$

An implication is that we can immediately use the result of part a to write the variance of output under the intervention policy as

$$\mathrm{E}_{t-1}\{\mathsf{y}_t\}^2 = \frac{\theta^2\left(\tilde{\eta}^2\sigma_\epsilon^2 + \delta^2\sigma_\nu^2\right)}{\left[\delta(1+\phi\theta) + \tilde{\eta}(\theta+\delta)\right]^2}.$$

To find the optimal value of the synthetic parameter $\tilde{\eta}$, we compute the derivative

$$\frac{\mathrm{dE}_{t-1}\{\mathsf{y}_t\}^2}{\mathrm{d}\tilde{\eta}} = 2\theta^2 \frac{\left[\delta(1+\phi\theta) + \tilde{\eta}(\theta+\delta)\right]\tilde{\eta}\sigma_\epsilon^2 - \left(\tilde{\eta}^2\sigma_\epsilon^2 + \delta^2\sigma_\nu^2\right)(\theta+\delta)}{\left[\delta(1+\phi\theta) + \tilde{\eta}(\theta+\delta)\right]^3}$$

and set it equal to zero. (We leave it to the student to check that the relevant second-order conditions are satisfied.) The resulting condition simplifies considerably, to

$$\delta(1+\phi\theta)\tilde{\eta}\sigma_{\epsilon}^{2}-(\theta+\delta)\delta^{2}\sigma_{\nu}^{2}=0,$$

which is readily solved to yield:

$$\tilde{\eta}=\frac{(\theta+\delta)\delta\sigma_{\nu}^{2}}{(1+\phi\theta)\sigma_{\epsilon}^{2}}.$$

Finally, using the definition of $\tilde{\eta}$ above, we find that the optimal monetary reaction parameter is

$$\Phi^{*}=\frac{(\theta+\delta)\delta}{(1+\phi\theta)}\left(\frac{\sigma_{\nu}^{2}}{\sigma_{\epsilon}^{2}}\right)-\eta.$$

It is instructive to consider how this optimal rule embodies the answer to part c above. Notice that Φ^{*} is monotonically increasing in the variance ratio $\sigma_{\nu}^{2}/\sigma_{\epsilon}^{2}$. The greater the variance of monetary compared with real shocks, the greater the tendency for intervention policy to "lean against the wind." As $\sigma_{\nu}^{2}/\sigma_{\epsilon}^{2}\to\infty$, $\Phi^{*}\to\infty$ as well, meaning that we have a fixed exchange rate: any rise in e over $\bar{\mathsf{e}}$, for example, elicits a large monetary contraction that drives e immediately back to its target level. Leaning against the wind is not always an optimal response, however. As $\sigma_{\nu}^{2}/\sigma_{\epsilon}^{2}\to 0$ (in which case real shocks are preponderant), $\Phi^{*}\to-\eta$ according to the preceding formula. It becomes optimal for intervention policy to accentuate rather than resist the exchange rate's movement. The solutions for the exchange rate, price of domestic output, and output levels (recall part a, above) show why this is so. A policy that sets $\Phi^{*}=-\eta$ effectively sets the interest semi-elasticity of money demand equal to zero. The solutions in part a show that in that case, the shock ϵ affects the exchange rate, but not output or domestic prices. The role of monetary policy is to ensure that, when world demand for domestic goods rises (say), the currency appreciates by enough to forestall any increase in output. That policy reduces the variance of output to zero.

4. (a) Proceeding as in section 9.5.2.2 in the book one finds that

$$\pi_t^{\mathrm{e}} = \frac{k - \mathrm{E}_{t-1}\{\lambda_t\}\omega}{\chi} = \frac{k-\omega}{\chi}$$

and

$$\pi_t = \pi_t^{\mathrm{e}} - \frac{[\lambda_t - \mathrm{E}_{t-1}\{\lambda_t\}]\,\omega}{1+\chi} + \frac{z_t}{1+\chi} = \pi_t^{\mathrm{e}} - \frac{(\lambda_t - 1)\,\omega}{1+\chi} + \frac{z_t}{1+\chi}.$$

Mean inflation is the same as when λ is known to equal 1, but the ex post uncertainty in policymaker type creates extra inflation variability.

(b) To calculate expected social loss, assume $\mathrm{Cov}(\lambda, z) = 0$ and compute

$$\begin{aligned}
\mathrm{E}_{t-1}\mathfrak{L}_t &= \mathrm{E}_{t-1}\left\{\left(-\frac{(\lambda_t-1)\,\omega}{1+\chi} + \frac{z_t}{1+\chi} - z_t - k\right)^2\right. \\
&\quad \left. + \chi\left(\frac{k-\omega}{\chi} - \frac{(\lambda_t-1)\,\omega}{1+\chi} + \frac{z_t}{1+\chi}\right)^2\right\} \\
&= k^2 + \frac{\sigma_\lambda^2\omega^2}{(1+\chi)^2} + \frac{\chi^2\sigma_z^2}{(1+\chi)^2} + \frac{(k-\omega)^2}{\chi} + \frac{\chi\sigma_\lambda^2\omega^2}{(1+\chi)^2} + \frac{\chi\sigma_z^2}{(1+\chi)^2} \\
&= k^2 + \frac{(k-\omega)^2}{\chi} + \frac{\sigma_\lambda^2\omega^2}{1+\chi} + \frac{\chi\sigma_z^2}{1+\chi}.
\end{aligned}$$

(c) When $\sigma_\lambda^2 = 0$, the clear solution is $\omega = k$ (the Walsh 1995 solution); but with λ unpredictable, there is a trade-off between reducing *mean* inflation by choosing a positive ω and raising the *variance* of inflation because the policymaker's preferences are random. From the first order condition for minimizing the expected loss in part b with respect to ω we can solve for ω as:

$$\omega = \frac{(1+\chi)k}{1+\chi(1+\sigma_\lambda^2)}.$$

Thus, greater uncertainty about the weight λ reduces the optimal ω below the benchmark value of k.

5. (a) (Note: In this problem the reader can simplify by setting $k = 0$, without substantively changing the nature of the exercise. However, the solution here is for an arbitrary k.) Taking π_t^e as given, differentiate $\mathfrak{L}_t$ with respect to π_t to solve for the one-shot-game equilibrium level of π_t. The first-order condition is $-\lambda_t + \pi_t = 0$, implying that

$$\pi_t = \lambda_t$$

in equilibrium. With rational expectations,

$$\pi_t^e = \mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\lambda_t = \tfrac{1}{2} \cdot 0 + \tfrac{1}{2} \cdot 2 = 1.$$

In part b we will need to know the expected loss in the discretionary one-shot-game equilibrium. Because $\pi_t^e = 1$,we can calculate it as

$$\mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{D}} = \mathrm{E}_{t-1}\left\{-\lambda_t\left(\lambda_t - 1 - k\right) + \tfrac{1}{2}\lambda_t^2\right\}$$

Since $\mathrm{E}_{t-1}\lambda_t^2 = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 4 = 2$, however, the preceding expression simplifies to

$$\mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{D}} = -2 + 1 + k + \tfrac{1}{2} \cdot 2 = k.$$

(b) The reaction function must satisfy

$$\tfrac{1}{2} \cdot \pi(0) + \tfrac{1}{2} \cdot \pi(2) = 0$$

and minimize

$$\mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{C}} = \tfrac{1}{2}\left\{\tfrac{1}{2}\pi(0)^2\right\} + \tfrac{1}{2}\left\{-2\left[\pi(2) - k\right] + \tfrac{1}{2}\pi(2)^2\right\}.$$

Substituting the constraint of zero expected inflation into the loss function, we write the problem to be solved as the unconstrained minimization of

$$\mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{C}} = \tfrac{1}{2}\left\{\tfrac{1}{2}\pi(2)^2\right\} + \tfrac{1}{2}\left\{-2\left[\pi(2) - k\right] + \tfrac{1}{2}\pi(2)^2\right\}.$$

The first-order condition for a maximum is

$$\frac{\mathrm{dE}_{t-1}\mathfrak{L}_t^{\mathrm{C}}}{\mathrm{d}\pi(2)} = \tfrac{1}{2}\pi(2) - 1 + \tfrac{1}{2}\pi(2) = 0,$$

implying that $\pi(0) = -1$ and $\pi(2) = 1$. Given this policy rule,

$$\begin{aligned}
\mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{C}} &= \tfrac{1}{2}\left\{\tfrac{1}{2}(-1)^2\right\} + \tfrac{1}{2}\left\{-2(1-k) + \tfrac{1}{2}(1)^2\right\} \\
&= \tfrac{1}{4} + (k-1) + \tfrac{1}{4} \\
&= k - \tfrac{1}{2} < \mathrm{E}_{t-1}\mathfrak{L}_t^{\mathrm{D}} = k.
\end{aligned}$$

(c) Suppose first that the central bank always reveals λ_t on date $t-1$ before π_t^e is set, so that $\pi_t^e = \lambda_t$. Its expected loss (from the standpoint of date $t-2$, when it itself doesn't yet know its value of λ_t) is

$$\mathrm{E}_{t-2}\mathfrak{L}_t^{\mathrm{YES}} = \tfrac{1}{2}\cdot 0 + \tfrac{1}{2}(2k+2) = k+1.$$

If it does not reveal λ_t then $\pi_t^e = 1$ and the expected loss is

$$\mathrm{E}_{t-2}\mathfrak{L}_t^{\mathrm{NO}} = \tfrac{1}{2}\cdot 0 + \tfrac{1}{2}[-2(1-k)+2] = k.$$

Since $\mathrm{E}_{t-2}\mathfrak{L}_t^{\mathrm{YES}} > \mathrm{E}_{t-2}\mathfrak{L}_t^{\mathrm{NO}}$, the central bank would prefer to precommit itself not to reveal λ_t before the public sets π_t^e. That is, the central bank prefers a system that mandates central-bank secrecy about its true preferences—what has sometimes been characterized as "monetary mystique." (This result is heavily dependent on the specific form of loss function assumed here.)