Foundations of International Macroeconomics[1]

Workbook[2]

Maurice Obstfeld, Kenneth Rogoff, and Gita Gopinath

Chapter 8 Solutions

1. First think about the *continuous-time* case. At time $t = 0$ the market believes the government's promise that it will peg the exchange rate at its $t = T$ level from time T onward. In that case, we can show that the exchange rate is *indeterminate*, so that the government's policy is not coherent (in that it does not tie down a unique market equilibrium). Let perfect-foresight equilibrium be described by the continuous-time Cagan model [eq. (70) on p. 559 of the book],

$$\mathsf{m}_t = \mathsf{e}_t - \eta\dot{\mathsf{e}}_t.$$

Let e_T^a be an arbitrary time-T exchange rate and suppose the market firmly expects that rate to prevail. Then the preceding Cagan equation, coupled with the terminal condition $\mathsf{e}_T = \mathsf{e}_T^a$, shows that the exchange rate path

$$\mathsf{e}_t = \frac{1}{\eta}\int_t^T \exp[(t-s)/\eta]\mathsf{m}_s \mathrm{d}s + \exp[(t-T)/\eta]\mathsf{e}_T^a$$

will equilibrate markets for $t \in [0, T]$.

Next consider the monetary authority's position at time T when confronted with this exchange rate path. The authority has no choice, in view of its vow of a constant exchange rate from T on, but to set the fundamental $\mathsf{m}_T = \mathsf{e}_T^a$ and to hold $\mathsf{m}_t = \mathsf{e}_T^a$ for all $t > T$. Why? Were the authority to act otherwise, the exchange rate would deviate from e_T^a at some point in the

[1]By Maurice Obstfeld (University of California, Berkeley) and Kenneth Rogoff (Princeton University).

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interval $[T, \infty)$, in violation of the government's initial pledge. In short, the authority must validate (ratify) any market expectation whatsoever. Any initial exchange rate can be an equilibrium because each is conditioned on a different expectation of what the (constant) money supply path will be from date T onward.

The situation is subtly different in *discrete time*, in which case the model is

$$\mathsf{m}_t = \mathsf{e}_t - \eta \left(\mathsf{e}_{t+1} - \mathsf{e}_t \right).$$

The basic reason discrete time makes a difference is that now, if the market firmly foresees a date T rate of e_T^a, there are *two* distinct ways the authority can fulfill its promise of a constant exchange rate from date T on (two alternatives which collapse to one in continuous time). First, the authority could still set $\mathsf{m}_t = \mathsf{e}_T^a$ for $t = T$ and for $t > T$, as in the continuous-time setting—in which case the exchange rate again is not uniquely determined. Alternatively, if the authority can commit not to adjust m_T fully to validate e_T^a, but instead only to set $\mathsf{m}_t = \mathsf{e}_T^a$ for t strictly greater than T only, we again get an exchange rate path constant at e_T^a starting on date T. In this second case, however, the exchange rate is uniquely determined. To fix ideas, suppose the authority can commit to a fixed value $\overline{\mathsf{m}}_T$ for date T. In that case, since the date $T + 1$ rate will be pegged at its date T level, the Cagan equation says that the exchange rate must satisfy

$$\mathsf{e}_T = \overline{\mathsf{m}}_T$$

on date T, a well-defined solution.

The distinction we make here actually does arise in practice. For example, the Maastricht Treaty on European Union (in effect) links to prior market values the "irrevocably fixed" bilateral currency conversion factors that will apply to member currencies when European economic and monetary union (EMU) starts on 4 January 1999. (EMU starts officially on Friday, 1 January 1999, but the first business day for the new European System of Central

Banks is Monday, 4 January.) Member currency bilateral rates must be fixed forever at their 31 December 1998 levels. This requirement need not lead to exchange-rate indeterminacy, however, even if participating national central banks do not intervene to set bilateral currency values on the last day of 1998. The reason is that the national central banks that will still be in operation through the end of 1998 have no automatic incentive to ratify market exchange rates on the last day before EMU. For a full discussion, see Maurice Obstfeld, "A strategy for launching the Euro," *European Economic Review* 42 (May 1998).

2. (a) Using the individual's budget constraint to substitute for C_t in the utility function, and taking derivatives with respect to B_{s+1} and M_s, one obtains the following first-order conditions:

$$B_{s+1}: \quad u'(C_s) = (1+r)\beta u'(C_{s+1}),$$

$$M_s: \quad \frac{u'(C_s)}{P_s}\left[1 - Yg'\left(\frac{M_s}{P_s}\right)\right] = \frac{\beta u'\left(C_{s+1}\right)}{P_{s+1}}.$$

Using the consumption Euler equation, we can rewrite the money demand equation as

$$Yg'\left(\frac{M_s}{P_s}\right) = \frac{i_{s+1}}{1+i_{s+1}}.$$

(b) If one assumes there are no speculative bubbles, the price level grows at the same gross rate $1+\mu$ as the nominal money supply. Steady-state real money balances thus are

$$\frac{\overline{M}}{P} = g'^{\,-1}\left[\frac{1}{Y}\left(1 - \frac{\beta}{1+\mu}\right)\right].$$

(c) With the new budget constraint, the first-order conditions become:

$$B_{s+1}: \quad u'(C_s)g\left(\frac{M_s}{P_s}\right) = (1+r)\beta u'(C_{s+1})g\left(\frac{M_{s+1}}{P_{s+1}}\right),$$

$$M_s: \quad \begin{aligned} & u'(C_s)\left[g\left(\frac{M_s}{P_s}\right)\frac{1}{P_s} - \frac{C_s}{g\left(\frac{M_s}{P_s}\right)}g'\left(\frac{M_s}{P_s}\right)\frac{1}{P_s}\right] \\ & = \beta u'(C_{s+1})\left[g\left(\frac{M_{s+1}}{P_{s+1}}\right)\frac{1}{P_{s+1}}\right]. \end{aligned}$$

One can then rewrite the money demand equation as

$$C_s \frac{g'\left(\frac{M_s}{P_s}\right)}{g\left(\frac{M_s}{P_s}\right)^2} = \frac{i_{s+1}}{1+i_{s+1}}.$$

(d) The analysis here parallels that in the text.

3. (a) The constraint follows by straightforward addition. Because domestic money is a nontraded asset, the present value of private and government spending must equal the present value of the economy's tradable resources, which in turn equals the sum of the present value of output and the economy's net foreign financial wealth.

(b) The answer does not change. The variable B_t in part a, the overall net foreign assets of the economy as a whole, equals the sum of domestic government and private-sector net assets, $B_t = B_t^{\mathrm{G}} + B_t^{\mathrm{P}}$ [recall eq. (7) from Chapter 3]. Equation (38) in Chapter 8 would change, however, in that $(1+r)B_t^{\mathrm{P}}$ rather than $(1+r)B_t$ would appear on its right-hand side. The last equation in footnote 26, p. 537 (the government budget constraint) would also differ, in that $(1+r)B_t^{\mathrm{G}}$ would be added to its right-hand side.

(c) Let us take the setup of Chapter 4, but with the services of money being the nontraded good and with $i_{s+1}/(1+i_{s+1})$ that good's date s price in terms of the tradable, consumption (recall section 8.3.3). When $\theta = 1$ (the Cobb-Douglas case), footnote 22, Chapter 4, tells us that

$$\Omega\left(C, \frac{M}{P}\right) = \frac{C^{\gamma}(M/P)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

and that

$$P_s^{\mathrm{C}} = \left(\frac{i_{s+1}}{1+i_{s+1}}\right)^{1-\gamma}.$$

Equation (26) of Chapter 4, translated to apply to the current setting, is the Euler equation for real consumption,

$$C_{s+1}^{\gamma}\left(\frac{M_{s+1}}{P_{s+1}}\right)^{1-\gamma} = \left[\frac{(1+r)P_s^{\mathrm{C}}}{P_{s+1}^{\mathrm{C}}}\right]^{\sigma}\beta^{\sigma}C_s^{\gamma}\left(\frac{M_s}{P_s}\right)^{1-\gamma}.$$

With $\theta = 1$, we also have that

$$\frac{M_s}{P_s} = \left(\frac{1-\gamma}{\gamma}\right)\left(P_s^{\mathrm{C}}\right)^{-1/(1-\gamma)}C_s$$

[eq. (40) in Chapter 8]. Substituting this relation into the Euler equation preceding it, we derive

$$C_{s+1} = \left(\frac{P_s^{\mathrm{C}}}{P_{s+1}^{\mathrm{C}}}\right)^{\sigma-1}(1+r)^{\sigma}\beta^{\sigma}C_s,$$

which parallels eq. (34) in Chapter 4 for the case $\theta = 1$. If you combine this equation with the intertemporal constraint derived in part a of this exercise, the result is

$$C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)}{\sum_{s=t}^{\infty}\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t^{\mathrm{C}}}{P_s^{\mathrm{C}}}\right)^{\sigma-1}}.$$

The definition of the consumption-based real interest rate (section 8.3.3) leads to the equivalent formula

$$C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)}{\sum_{s=t}^{\infty}\left[\prod_{v=t+1}^{s}(1+r_v^{\mathrm{C}})\right]^{\sigma-1}\beta^{\sigma(s-t)}}.$$

When $\sigma = 1$, equilibrium consumption is simply

$$C_t = (1-\beta)\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)\right],$$

as in a model with no money.

(d) In the more general case in which $\theta \neq 1$, the consumption Euler equation is

$$C_{t+1} = \left(\frac{P_t^{\mathrm{C}}}{P_{t+1}^{\mathrm{C}}}\right)^{\sigma-\theta}(1+r)^{\sigma}\beta^{\sigma}C_t,$$

which follows directly from eq. (34) in Chapter 4. Solving as in part c leads to

$$C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)}{\sum_{s=t}^{\infty}\left[(1+r)^{\sigma-1}\beta^{\sigma}\right]^{s-t}\left(\frac{P_t^{\mathrm{C}}}{P_s^{\mathrm{C}}}\right)^{\sigma-\theta}}.$$

4. For a time-varying real interest rate in eq. (59) of Chapter 8, the consumption Euler equation is

$$\frac{P_{s-1}}{P_s}u'(C_s) = (1+r_s)\frac{P_s}{P_{s+1}}\beta u'(C_{s+1}).$$

Making use of the Fisher parity equation $1 + i_{s+1} = (1 + r_{s+1})(P_{s+1}/P_s)$, we divide both sides by $1 + r_s$ to derive

$$\frac{u'(C_s)}{1+i_s} = (1+r_{s+1})\beta\frac{u'(C_{s+1})}{1+i_{s+1}}.$$

5. Under the revised fundamentals process, we still have eq. (83), p. 571, but now

$$\mathrm{E}_t\left\{G'(\mathsf{k}_t)\mathrm{d}\mathsf{k}_{t+h}\right\} = \mu h G'(\mathsf{k}_t)$$

rather than 0, while it is still true that $\mathrm{E}_t\left\{(\mathrm{d}\mathsf{k}_{t+h})^2\right\} \approx hv^2$ (because terms multiplied by h^2 and $h^{3/2}$ disappear as $h \to 0$, being of order greater than h). Thus,

$$\mathrm{E}_t\mathrm{d}G(\mathsf{k}_{t+h}) = \mu h G'(\mathsf{k}_t) + \frac{hv^2}{2}G''(\mathsf{k}_t)$$

in the limit as $h \to 0$. Plugging this result into eq. (82) on p. 571 leads to

$$G(\mathsf{k}) = \mathsf{k} + \eta\mu G'(\mathsf{k}) + \frac{\eta v^2}{2} G''(\mathsf{k})$$

as the differential equation any exchange rate solution must satisfy [cf. eq. (84), p. 572]. A general solution is of the form

$$G(\mathsf{k}) = \mathsf{k} + \alpha + b_1 \exp(\lambda_1 \mathsf{k}) + b_2 \exp(\lambda_2 \mathsf{k})$$

where the b's are arbitrary constants. Because internal consistency requires that

$$\mathsf{k} + \alpha + b_1 \exp(\lambda_1 \mathsf{k}) + b_2 \exp(\lambda_2 \mathsf{k})$$

$$= \mathsf{k} + \eta\mu \left[1 + \lambda_1 b_1 \exp(\lambda_1 \mathsf{k}) + \lambda_2 b_2 \exp(\lambda_2 \mathsf{k})\right]$$

$$+ \frac{\eta v^2}{2} \left[(\lambda_1)^2 b_1 \exp(\lambda_1 \mathsf{k}) + (\lambda_2)^2 b_2 \exp(\lambda_2 \mathsf{k})\right],$$

$\alpha = \eta\mu$ and λ_1 and λ_2 are the two roots of the quadratic equation

$$\frac{\eta v^2}{2} \lambda^2 + \eta\mu\lambda - 1 = 0.$$

The *particular* target zone solution still satisfies the "smooth pasting" conditions $G'(\mathsf{k}) = 0$ at the top and bottom of the band. The argument is the same as in section 8.5.4, because at the edges of the zone $\mathrm{E}_t \{\mathsf{dk}_{t+h}\}$ still changes discontinuously even when $\mu \neq 0$ (movements that would drive the exchange rate out of the band suddenly are prohibited).

6. (a) Intuitively, spending a domestic currency unit on the least-cost consumer basket such that $C = \Omega(C_1, ..., C_N) = 1$ yields $1/P$ units of C. Alternatively, spending the currency unit on any individual commodity j yields $1/p_j$ units of that good, each increasing C by $\Omega_j \equiv \partial C / \partial C_j$. At an optimum these two uses of the currency unit must have the same impact on C, so that the equality

$$\frac{1}{P} = \frac{1}{p_j} \frac{\partial C}{\partial C_j}$$

follows. Alternatively and more formally, consider the optimization problem that defines the CPI P, which is to minimize the expenditure $P = \sum_{j=1}^{N} p_j C_j$ subject to $\Omega(C_1, ..., C_N) = 1$. One way to solve this problem is to set up the Lagrangian

$$\mathcal{L} = \sum_{j=1}^{N} p_j C_j - \lambda \left[\Omega(C_1, ..., C_N) - 1 \right].$$

The first-order optimality conditions are (for all j):

$$\frac{\partial \mathcal{L}}{\partial C_j} = p_j - \lambda \Omega_j = 0.$$

Since $\Omega(C_1, ..., C_N)$ is linear homogeneous, we have

$$\sum_{j=1}^{N} \Omega_j C_j = \Omega(C_1, ..., C_N) = 1$$

at the optimum. Thus, multiplying the preceding first-order condition by C_j and summing over all j, we derive

$$P = \sum_{j=1}^{N} p_j C_j = \lambda \sum_{j=1}^{N} \Omega_j C_j = \lambda.$$

This equality, however, allows us to write the first-order condition as

$$\Omega_j = \frac{\partial C}{\partial C_j} = \frac{p_j}{P}.$$

(b) The difference between the ex post real return on a nominal Home-currency bond and that on a nominal Foreign-currency bond is

$$\frac{(1 + i_{t+1}) P_t}{P_{t+1}} - \frac{(1 + i_{t+1}^*) P_t^*}{P_{t+1}^*},$$

where P and P^* are the consumption-based price levels measured in Home and Foreign currency, respectively. Observe that these consumption-based price levels will be linked by purchasing power parity (absent trade barriers), because they simply measure the price of a single basket of commodities in two currencies. Thus we may write the preceding real return differential as

$$\frac{(1+i_{t+1})P_t}{P_{t+1}} - \frac{(1+i^*_{t+1})P_t/\mathcal{E}_t}{P_{t+1}/\mathcal{E}_{t+1}}.$$

Equation (104) in Chapter 8 (covered interest parity) allows us to write the foregoing difference as

$$\frac{(1+i_{t+1})P_t}{P_{t+1}} - \frac{(1+i_{t+1})P_t/\mathcal{F}_t}{P_{t+1}/\mathcal{E}_{t+1}},$$

and eq. (116) therefore implies that

$$\mathrm{E}_t\left\{\left[\frac{(1+i_{t+1})P_t}{P_{t+1}} - \frac{\mathcal{E}_{t+1}(1+i_{t+1})P_t/\mathcal{F}_t}{P_{t+1}}\right]\frac{u'(C_{t+1})}{u'(C_t)}\right\} = 0.$$

Factoring out the term $(1+i_{t+1})\,P_t/\mathcal{F}_t$, which is date t information, we are left with

$$0 = \mathrm{E}_t\left\{\left(\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}}\right)\frac{u'(C_{t+1})}{u'(C_t)}\right\}.$$

(c) Substitute $\mathcal{E}_{t+1}P^*_{t+1} = P_{t+1}$ into the previous equation and multiply by $-1/\mathcal{F}_t$ to get

$$0 = \mathrm{E}_t\left\{\left(\frac{\frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}}}{P^*_{t+1}}\right)\frac{u'(C_{t+1})}{u'(C_t)}\right\}.$$

Observe that this condition is perfectly symmetrical to the one derived in part b from the Home investor's perspective. In the risk neutral case, the marginal utility of consumption is constant, and therefore we have that

$$\mathrm{E}_t\left\{\frac{\frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}}}{P^*_{t+1}}\right\} = 0 = \mathrm{E}_t\left\{\frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}}\right\}.$$

Plainly Siegel's paradox does not apply.

(d) The result follows immediately from part a, where it was shown that $\partial C/\partial Cj = p_j/P$.

(e) The condition that consumers equate their marginal rates of substitution to relative prices implies that

$$\frac{C_{\mathrm{X},t}}{C_{\mathrm{Y},t}} = \left(\frac{\gamma}{1-\gamma}\right)\frac{\mathcal{E}_t p^*_{\mathrm{Y},t}}{p_{\mathrm{X},t}},$$

so that we can express the spot exchange rate as

$$\mathcal{E}_t = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{p_{\mathrm{X},t}C_{\mathrm{X},t}}{p^*_{\mathrm{Y},t}C_{\mathrm{Y},t}}\right).$$

In a perfectly pooled risk-sharing equilibrium, $C_{\mathrm{X},t} = C^*_{\mathrm{X},t} = X_t/2$ and $C_{\mathrm{Y},t} = C^*_{\mathrm{Y},t} = Y_t/2$. Moreover, using the (binding) cash-in-advance constraints for the two currencies, $M_t = p_{\mathrm{X},t}X_t$ and $M^*_t = p^*_{\mathrm{Y},t}Y_t$, we can express the equation for the spot exchange rate as

$$\mathcal{E}_t = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{M_t}{M^*_t}\right).$$

Using the result in part d:

$$\mathcal{F}_t = \frac{\mathrm{E}_t\left\{\dfrac{\mathcal{E}_{t+1}u_j(C_{t+1})}{p_{j,t+1}}\right\}}{\mathrm{E}_t\left\{\dfrac{u_j(C_{t+1})}{p_{j,t+1}}\right\}}, \quad j = \mathrm{X}, \mathrm{Y}.$$

In combination with the cash-in-advance constraints and the preceding relationship between the spot exchange rate and money supplies, we have, for $j = \mathrm{X}$,

$$\mathcal{F}_t = \left(\frac{1-\gamma}{\gamma}\right)\frac{\mathrm{E}_t\left\{\dfrac{C_{\mathrm{X},t+1}}{M^*_{t+1}}u_{\mathrm{X}}(C_{t+1})\right\}}{\mathrm{E}_t\left\{\dfrac{C_{\mathrm{X},t+1}}{M_{t+1}}u_{\mathrm{X}}(C_{t+1})\right\}}.$$

If money and output shocks have statistically independent distributions, however, we may factor out the terms in consumption above and write

$$\begin{aligned}
\mathcal{F}_t &= \left(\frac{1-\gamma}{\gamma}\right)\frac{\mathrm{E}_t\left\{\dfrac{1}{M^*_{t+1}}\right\}\mathrm{E}_t\left\{C_{\mathrm{X},t+1}u_{\mathrm{X}}(C_{t+1})\right\}}{\mathrm{E}_t\left\{\dfrac{1}{M_{t+1}}\right\}\mathrm{E}_t\left\{C_{\mathrm{X},t+1}u_{\mathrm{X}}(C_{t+1})\right\}} \\
&= \left(\frac{1-\gamma}{\gamma}\right)\frac{\mathrm{E}_t\left\{\dfrac{1}{M^*_{t+1}}\right\}}{\mathrm{E}_t\left\{\dfrac{1}{M_{t+1}}\right\}}.
\end{aligned}$$

(f) The result in part a and the cash-in-advance constraint imply that

$$M = p_{\mathrm{X}}C_{\mathrm{X}} = PC_{\mathrm{X}}\frac{\partial C}{\partial C_{\mathrm{X}}}.$$

From part e we therefore obtain the "risk neutral" forward exchange rate, eq. (107) in Chapter 8, by again using the assumption that outputs and monies are independently distributed random variables:

$$\begin{aligned}
\mathcal{F}_t &= \frac{\mathrm{E}_t\left\{\left(\dfrac{1-\gamma}{\gamma}\right)\left(\dfrac{M_{t+1}}{M^*_{t+1}}\right)\dfrac{1}{M_{t+1}}\right\}}{\mathrm{E}_t\left\{\dfrac{1}{M_{t+1}}\right\}} \\
&= \frac{\mathrm{E}_t\left\{\mathcal{E}_{t+1}\dfrac{1}{M_{t+1}}\right\}\mathrm{E}_t\left\{C_{\mathrm{X},t+1}\dfrac{\partial C_{t+1}}{\partial C_{\mathrm{X},t+1}}\right\}}{\mathrm{E}_t\left\{\dfrac{1}{M_{t+1}}\right\}\mathrm{E}_t\left\{C_{\mathrm{X},t+1}\dfrac{\partial C_{t+1}}{\partial C_{\mathrm{X},t+1}}\right\}} \\
&= \frac{\mathrm{E}_t\left\{\mathcal{E}_{t+1}\left(\dfrac{C_{\mathrm{X},t+1}}{M_{t+1}}\right)\dfrac{\partial C_{t+1}}{\partial C_{\mathrm{X},t+1}}\right\}}{\mathrm{E}_t\left\{\left(\dfrac{C_{\mathrm{X},t+1}}{M_{t+1}}\right)\dfrac{\partial C_{t+1}}{\partial C_{\mathrm{X},t+1}}\right\}} \\
&= \frac{\mathrm{E}_t\left\{\mathcal{E}_{t+1}/P_{t+1}\right\}}{\mathrm{E}_t\left\{1/P_{t+1}\right\}}.
\end{aligned}$$

Why is there no risk premium? Here, the marginal utility of consumption is conditionally uncorrelated with the exchange rate when output and money shocks are independent. But if that correlation is zero, there is no foreign exchange risk premium. The last two results asked for in this part of the exercise follow from parts c and d above.

7. (a) The two-week average exchange rate series, sampled biweekly, does not follow a random walk even though the weekly exchange rate does. This can be shown very easily:

$$\begin{aligned}
\mathrm{E}_t\left\{\widetilde{\mathcal{E}}_{t+2}\right\} &= \mathrm{E}_t\left\{\tfrac{1}{2}(\mathcal{E}_{t+2}+\mathcal{E}_{t+1})\right\} \\
&= \mathrm{E}_t\left\{\tfrac{1}{2}(\mathcal{E}_{t+1}+\epsilon_{t+2}+\mathcal{E}_{t+1})\right\} \\
&= \mathrm{E}_t\left\{\tfrac{1}{2}(2\mathcal{E}_{t}+2\epsilon_{t+1}+\epsilon_{t+2})\right\} \\
&= \mathcal{E}_t \neq \widetilde{\mathcal{E}}_t \equiv \tfrac{1}{2}(\mathcal{E}_t+\mathcal{E}_{t-1}).
\end{aligned}$$

(b) In this case the problem does not arise. As long as the weekly exchange rate follows a random walk, a series formed by taking point-sample observations every other week also follows a random walk.

8. (a) The weekly series of two-week prediction errors is serially correlated. Let us suppose that the spot rate follows a random walk, $\mathsf{e}_t = \mathsf{e}_{t-1} + \eta_t$, where η_t is a serially uncorrelated white noise disturbance term. Then

$$\mathrm{Cov}\left\{\mathsf{e}_{t+2}-\mathsf{f}_{t,2}, \mathsf{e}_{t+3}-\mathsf{f}_{t+1,2}\right\} = \mathrm{E}\left\{(\mathsf{e}_{t+2}-\mathsf{e}_t)(\mathsf{e}_{t+3}-\mathsf{e}_{t+1})\right\}.$$

The preceding equality follows from the definition of covariance and the two assumptions that $\mathsf{f}_{t,2} = \mathrm{E}_t\mathsf{e}_{t+2}$ and that the spot exchange rate follows a random walk. Since $\mathsf{e}_{t+2}-\mathsf{e}_t = \eta_{t+2}+\eta_{t+1}$ and $\mathsf{e}_{t+3}-\mathsf{e}_{t+1} = \eta_{t+3}+\eta_{t+2}$, the preceding equality can be written as

$$\begin{aligned}
\mathrm{Cov}\left\{\mathsf{e}_{t+2}-\mathsf{f}_{t,2}, \mathsf{e}_{t+3}-\mathsf{f}_{t+1,2}\right\} &= \mathrm{E}\left\{(\eta_{t+2}+\eta_{t+1})(\eta_{t+3}+\eta_{t+2})\right\} \\
&= \mathrm{E}\left\{\eta_{t+2}^2\right\} \neq 0.
\end{aligned}$$

The reason for the nonzero covariance is that temporally adjacent overlapping multiperiod forward-rate errors share common innovations. They share innovations because the maturity of the forward contract (two weeks) is longer than the sampling interval (one week). We can show through similar steps that for $j > 1$,

$$\mathrm{Cov}\left\{\mathsf{e}_{t+2} - \mathsf{f}_{t,2}, \mathsf{e}_{t+j+2} - \mathsf{f}_{t+j,2}\right\} = 0.$$

(b) A series sampled biweekly would not be serially correlated. This result follows easily from the argument of part a. In this case there are no overlapping multiperiod forecast errors.

(c) We outline how a General Method of Moments (GMM) estimator could be used to test the hypothesis that $\mathrm{E}_t(\mathsf{e}_{t+2} - \mathsf{f}_{t,2}) = 0$. Define the two-period-ahead forward-rate forecast error as $\epsilon_{t+2,2} \equiv \mathsf{e}_{t+2} - \mathsf{f}_{t,2}$. The null hypothesis states that the forward rate is equal to the conditional expectation of the two-period-ahead spot rate, and is therefore the best (most efficient) unbiased predictor of the spot rate. This property implies that $\epsilon_{t+2,2}$ will be uncorrelated with any information available at time t. One possible way to test this implication is to run the following regression [which is similar to equation (110) in the text],

$$\mathsf{e}_{t+2} - \mathsf{e}_t = a_0 + a_1\left(\mathsf{f}_{t,2} - \mathsf{e}_t\right) + \epsilon_{t+2,2}, \tag{1}$$

where the difference $\mathsf{f}_{t,2} - \mathsf{e}_t$ is the forward premium on date t. The forward-market "efficiency" test asks whether one can reject the joint null hypothesis that $a_0 = 0$ and $a_1 = 1$. Under the null hypothesis, $\epsilon_{t+2,2}$ equals the forward-rate forecast error and so the orthogonality conditions $\mathrm{E}\{\epsilon_{t+2,2}\} = 0$ and $\mathrm{E}\{(\mathsf{f}_{t,2} - \mathsf{e}_t)\,\epsilon_{t+2,2}\} = 0$ are satisfied for $a_0 = 0$ and $a_1 = 1$. The specification implies that the efficient GMM estimator of a_0 and a_1 is the same as the ordinary least squares (OLS) estimator based on all the available (weekly) observations of spot and two-week-ahead forward rates. However the usual OLS standard errors would not be appropriate. When we use the weekly series of two-week-ahead prediction errors, we face a problem of serial correlation in $\epsilon_{t+2,2}$ (as discussed in part a).

To describe the GMM estimator, define the coefficient column vector $\boldsymbol{\theta} = [a_0 \;\; a_1]'$, along with the vectors

$$\mathbf{x}_t \equiv \begin{bmatrix} 1 \\ \mathsf{f}_{t,2} - \mathsf{e}_t \end{bmatrix}$$

and

$$\mathbf{w}_t(\boldsymbol{\theta}) \equiv \mathbf{x}_t \cdot \epsilon_{t+2,2}(\boldsymbol{\theta}) = \begin{bmatrix} \epsilon_{t+2,2}(\boldsymbol{\theta}) \\ (\mathsf{f}_{t,2} - \mathsf{e}_t)\, \epsilon_{t+2,2}(\boldsymbol{\theta}) \end{bmatrix}.$$

By eq. (1) above,

$$\epsilon_{t+2,2}(\boldsymbol{\theta}) \equiv \mathsf{e}_{t+2} - \mathsf{e}_t - [a_0 + a_1 (\mathsf{f}_{t,2} - \mathsf{e}_t)]\,.$$

The efficient GMM estimator (EGMM) for a sample of size T is derived as

$$\hat{\boldsymbol{\theta}}_{\mathrm{EGMM}}(\hat{\boldsymbol{\Phi}}) = \mathrm{argmin}_{\boldsymbol{\theta}} \; \bar{\mathbf{w}}(\boldsymbol{\theta})'\hat{\boldsymbol{\Phi}}^{-1}\bar{\mathbf{w}}(\boldsymbol{\theta}),$$

where

$$\bar{\mathbf{w}}(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{w}_t(\boldsymbol{\theta}),$$

$$\boldsymbol{\Phi} = \sum_{j=-\infty}^{\infty}\boldsymbol{\Gamma}_j,$$

$$\boldsymbol{\Gamma}_j = \mathrm{E}\left\{\mathbf{w}_t(\boldsymbol{\theta})\mathbf{w}_{t-j}(\boldsymbol{\theta})'\right\},$$

and $\hat{\boldsymbol{\Phi}}$ is a consistent estimate of $\boldsymbol{\Phi}$. Under standard regularity conditions, the estimate $\hat{\boldsymbol{\theta}}_{\mathrm{EGMM}}$ is asymptotically normal with mean θ. One can estimate the asymptotic covariance matrix of $\hat{\boldsymbol{\theta}}_{\mathrm{EGMM}}$ by

$$T\left(\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\right)^{-1}\hat{\boldsymbol{\Phi}}\left(\sum_{t=1}^{T}\mathbf{x}_t\mathbf{x}_t'\right)^{-1}.$$

Under the null hypothesis the coefficient vector is $\boldsymbol{\theta}_0 = [0, 1]'$. One can perform a Wald/Likelihood Ratio/Lagrange Multiplier test of the joint null hypothesis. (For details on GMM estimation and hypothesis testing, see R. Davidson and J. G. Mackinnon, *Estimation and Inference in Econometrics*, Oxford, 1993.)