Foundations of International Macroeconomics[1]

Workbook[2]

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Chapter 6 Solutions

1. (a) We look at the symmetric efficient incentive-compatible contract. That contract maximizes an equally-weighted average of Home and Foreign expected utility,

$$\mathrm{E}\left\{u(C)\right\}+\mathrm{E}\left\{u(C^{*})\right\},$$

subject to the constraints

$$-\eta Y^{*}\leq P(\epsilon)\leq\eta Y,$$

which must hold for all N possible realizations of the shock ϵ. The Lagrangian for the contracting problem is

$$\begin{aligned}\mathcal{L} &= \max_{P(\epsilon)}\sum_{1=1}^{N}\pi(\epsilon_i)\left\{u\left[\bar{Y}+\epsilon_i-P(\epsilon_i)\right]+u\left[\bar{Y}-\epsilon_i+P(\epsilon_i)\right]\right\}\\ &\quad-\sum_{1=1}^{N}\lambda(\epsilon_i)\left[P(\epsilon_i)-\eta(\bar{Y}+\epsilon_i)\right]-\sum_{1=1}^{N}\mu(\epsilon_i)\left[-P(\epsilon_i)-\eta(\bar{Y}-\epsilon_i)\right].\end{aligned}$$

The first-order condition with respect to $P(\epsilon_i)$ is

$$\pi(\epsilon_i)\left\{-u'\left[C(\epsilon_i)\right]+u'\left[C^{*}(\epsilon_i)\right]\right\}-\lambda(\epsilon_i)+\mu(\epsilon_i)=0, \qquad (1)$$

and the complementary slackness conditions are

$$\lambda(\epsilon_i)\left[\eta(\bar{Y}+\epsilon_i)-P(\epsilon_i)\right]=0,$$

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$$\mu(\epsilon_i)\left[\eta(\bar{Y}-\epsilon_i)+P(\epsilon_i)\right]=0.$$

Despite the forbidding formalism of the Kuhn-Tucker conditions, the solution to the problem can be characterized rather simply. Consider the solution in states where the incentive constraints do not bind, that is when $\lambda(\epsilon_i)=\mu(\epsilon_i)=0$. Across these states, the first-order condition (1) reduces to $u'[C(\epsilon_i)]=u'[C^*(\epsilon_i)]$, and so $C(\epsilon_i)=C^*(\epsilon_i)$. There is therefore a range $[-e,e]$ such that inside this interval $C=C^*$. The bound e is easily found as the largest ϵ such that $\epsilon\leq\eta\left(\bar{Y}+\epsilon\right)$, implying

$$e=\frac{\eta}{1-\eta}\bar{Y}. \tag{2}$$

(b) For $\epsilon>e$, the incentive constraint $P(\epsilon_i)\leq\eta(\bar{Y}+\epsilon_i)$ prevents full insurance, so $P(\epsilon_i)=e+\eta(\epsilon_i-e)$ [where we have substituted for $\bar{Y}$ from equation (2)]. Since the equilibrium is symmetric, for $\epsilon<-e$, $P(\epsilon_i)=-e+\eta(\epsilon_i+e)$.

To graph the payments schedule that the contract implies, put $P(\epsilon)$ on the vertical axis and ϵ on the horizontal axis. The payments schedule passes through the origin, has slope 1 over $[-e,e]$, and has slope η outside that interval. See figure 6.1.

2. This problem is completely parallel to the problem in the text. There are only two differences: the zero-profit condition for lenders is now

$$\sum_{1=1}^{N}\pi(\epsilon_i)P(\epsilon_i)=(1+r)D,$$

and $P(\epsilon_i)\geq 0$ must obtain (because the country, by assumption, cannot receive any payments in period 2). It is easy to see that the country will not in general be able to attain as much insurance in this environment as in the two-way payment environment of the text. The basic problem is that the amounts lenders must collect are larger here because the country's net second-period payments must be positive. (The country's second-period income is also higher as a result of prior investment, but as long as $\eta < 1$, the former effect dominates.) For example, in the case of pure insurance contracts (as in the text), the necessary condition for full insurance to be feasible is

$$\bar{\epsilon} \leq \eta(\bar{Y} + \bar{\epsilon})$$

or

$$\bar{\epsilon} \leq \frac{\eta}{1-\eta}\bar{Y}.$$

Here, however, the corresponding condition is

$$\bar{\epsilon} + (1+r)D \leq \eta\left[\bar{Y} + \bar{\epsilon} + (1+r)D\right]$$

[because the contract with $P(\epsilon) = \epsilon + (1+r)D$, which yields full insurance, satisfies the zero-profit condition, and satisfies $P(\epsilon) \geq 0, \forall\epsilon$, is incentive-compatible only if the preceding inequality holds]. The preceding inequality is equivalent to

$$\bar{\epsilon} + (1+r)D \leq \frac{\eta}{1-\eta}\bar{Y}. \tag{3}$$

Since the minimum amount the country needs to borrow to achieve full insurance while keeping its second-period payments nonnegative is $-\underline{\epsilon}/(1+r) > 0$, full insurance therefore is feasible if and only if

$$\bar{\epsilon} - \underline{\epsilon} \leq \frac{\eta}{1-\eta}\bar{Y}.$$

Clearly, given that $-\underline{\epsilon} > 0$, the constraint under pure insurance contracts is less likely to be binding than in the case where insurance is achieved through an output-indexed borrowing contract. [Compare eq. (3) with the last term in footnote 8 of the chapter.]

(a) The reference to a "given" D in the exercise obviously refers to a level D small enough that the country can be compelled to make nonnegative payments with an expected value of $(1+r)D$—creditors would never agree to lend a larger amount in the first place! Qualitatively, the equilibrium is the same as in section 6.1.1 of the text. Over a range of ϵ below a cutoff value e, $\mathrm{d}P(\epsilon)/\mathrm{d}\epsilon = 1$, while above e, $\mathrm{d}P(\epsilon)/\mathrm{d}\epsilon = \eta$. For the optimal level of D, which now is generally smaller than $D = -\underline{\epsilon}/(1+r)$, $P(\underline{\epsilon}) = 0$. The optimal choice of D minimizes the expected value of the repayments the country must make.

(b) The country is clearly worse off if it only has access only to equity contracts, since in effect it must be able to commit to repayments with a positive expected value instead of repayments with a zero expected value.

3. Consumption in each period is given by

$$C_t = F(D_t) + \epsilon_t - P(\epsilon_t), \tag{4}$$

where $P(\epsilon_t) \geq 0$ and $D_t \geq 0$. The nonnegativity constraint on $P(\epsilon_t)$ reflects the assumption that it is not feasible to write and to enforce contracts that require insurers to indemnify the sovereign after the realization of a bad state of the world. The nonnegativity constraint on D_t reflects the assumption that the country cannot lend abroad.

Since we assume a competitive lending market, the risk-neutral foreign lenders receive an expected return of $1 + r$ in equilibrium, that is,

$$\sum_{\epsilon_t} \pi(\epsilon_t) P^{\mathrm{e}}_{t-1}(\epsilon_t) = (1+r)D_t, \tag{5}$$

where $P^{\mathrm{e}}_{t-1}(\epsilon_t)$ is the amount of debt servicing that the lenders in period $t-1$ expect to receive in period t as a function of the realization ϵ_t.

(a) Suppose that the country can commit itself in period $t-1$ to a payment schedule in period t, given by $P_{t-1}(\epsilon_t)$. In that case the country would determine the lenders' expectations of the actual level of debt servicing:

$$P^{\mathrm{e}}_{t-1}(\epsilon_t) = P_{t-1}(\epsilon_t). \tag{6}$$

Because the analysis assumes ϵ_t to be stationary, the country's choices are time invariant. The optimal $P(\epsilon_t)$ and D can be computed by maximizing

$\mathrm{E}_{t-1}u\left(C_t\right)$ subject to the constraints given by eqs. (4), (5), and (6). The Lagrangian for the country's problem is:

$$\mathcal{L}=\sum_{\epsilon_t}\pi(\epsilon_t)u\left[F(D)+\epsilon_t-P\left(\epsilon_t\right)\right]+\lambda\left[\sum_{\epsilon_t}\pi\left(\epsilon_t\right)P\left(\epsilon_t\right)-(1+r)\,D\right].$$

Taking derivatives with respect to $P(\epsilon_t)$ and D we obtain

$$u'\left[C(\epsilon_t)\right]=\lambda$$

for all ϵ_t and

$$F'(D)=1+r.$$

Combining the preceding two equations with (4) and (5), we obtain the critical values for $P\left(\epsilon_t\right)$ and D:

$$D^{*}=\max\left\{\tilde{D},\frac{e-\underline{\epsilon}}{1+r}\right\},\tag{7}$$

$$P\left(\epsilon_t\right)=\epsilon_t-e+(1+r)D^{*}.\tag{8}$$

Equations (7) and (8) indicate that the country, by irrevocably committing not to repudiate, is able to invest efficiently, thereby maximizing expected consumption. It also achieves efficient risk shifting. Equation (8) gives the debt-servicing commitment $P\left(\epsilon_t\right)$. It calls for adding the difference, which can be positive or negative, between the realization of ϵ_t and $e=\mathrm{E}_{t-1}\left\{\epsilon_t\right\}$ to repayment of loans D^{*} at the interest rate r. Equation (7) gives an amount of borrowing that allows investment up to a point where the marginal product of capital equals $1+r$, yet is also sufficient for lenders to prepay the indemnity associated with the worst possible state of the world (equal to the discounted value of the difference between e and $\underline{\epsilon}$). Why? Consumption is independent of the state of the world; by eq. (4) it is stabilized at

$$C_t=\bar{C}=F(\tilde{D})-(1+r)\tilde{D}+e$$

(for all t). But if $\tilde{D}$ is less than $(e-\underline{\epsilon})/(1+r)$, efficient risk shifting with nonnegative country payments to creditors requires borrowing beyond the minimum level $\tilde{D}$ necessary for efficient investment. [Recall that for $D^* > \tilde{D}$, $F(\tilde{D}) - (1+r)\tilde{D} = F(D^*) - (1+r)D^*$, so consumption remains at $\bar{C}$ for $D^* > \tilde{D}$.]

We next consider the conditions under which it is possible to sustain the above contract as a trigger-strategy equilibrium where the only penalty to default is that the country is excluded from all future borrowing. Suppose that on date t the country considers default. Its short-run gain is the extra utility on date t from avoiding repayment,

$$\text{Gain}(\epsilon_t) = u\left[F\left(D^*\right) + \epsilon_t\right] - u\left(\bar{C}\right).$$

The date t cost associated with default is

$$\text{Cost} = \sum_{s=t+1}^{\infty} \beta^{s-t} u(\bar{C}) - \text{E}_t \left\{ \sum_{s=t+1}^{\infty} \beta^{s-t} u(\epsilon_t) \right\}.$$

Since the economy is stationary we can drop the time subscript and rewrite the cost as

$$\text{Cost} = \frac{\beta}{1-\beta}\left[u(\bar{C}) - \text{E}u(\epsilon)\right].$$

Since the utility function is strictly concave and since domestic investment is profitable, $u(\bar{C}) > u(\text{E}\epsilon) > \text{E}u(\epsilon)$. There is therefore a positive cost to default. The commitment contract is sustainable in all states of nature (and on all dates) only if

$$\text{Gain}(\bar{\epsilon}) \leq \text{Cost}.$$

that is, when

$$u\left[F\left(D^*\right) + \bar{\epsilon}\right] - u\left(\bar{C}\right) \leq \frac{\beta}{1-\beta}\left[u(\bar{C}) - \text{E}u(\epsilon)\right].$$

[For more details and analysis of the case where the parameters of the model are such that trigger-strategy expectations cannot support the efficient allocation, refer to Herschel I. Grossman and John B. Van Huyck, "Sovereign debt as a contingent claim: Excusable default, repudiation, and reputation," *American Economic Review* 78 (December 1988): 1088-97.]

(b) Recall the discussion of section 6.2.2. in the text.

(c) Recall the discussion of section 6.1.2.4 in the text.

4. (a) If E_2 is very large—specifically, if $E_2 \geq (1+r)(\bar{I}-Y_1)$, where $\bar{I}$ is the first-best efficient investment level defined by eq. (44) in Chapter 6—then entrepreneurs can finance their investments with noncontingent loans such that $P(Z) = P(0)$, and the moral hazard problem disappears. So assume that $E_2 < (1+r)(\bar{I}-Y_1)$. With observable second-period endowment E_2, the agent's maximization problem becomes

$$\begin{aligned} \mathrm{E}C_2 &= \pi(I)\left[Z - P(Z)\right] - \left[1-\pi(I)\right]P(0) + (1+r)L + E_2 \\ &= \pi(I)\left[Z - P(Z)\right] - \left[1-\pi(I)\right]P(0) + (1+r)(Y_1 + D - I) + E_2, \end{aligned}$$

and the bankruptcy constraint becomes

$$P(0) \leq E_2.$$

By the same logic as in the text, this constraint must hold with equality—if it did not, the gap between $P(Z)$ and $P(0)$ would be unnecessarily large, suboptimally distorting investment. Given these modifications, the incentive compatibility condition becomes

$$\pi'(I)\left\{Z - \left[P(Z) - E_2\right]\right\} = 1 + r,$$

implying that the **IC** curve of the text is given by

$$P(Z) - E_2 = Z - \frac{1+r}{\pi'(I)}.$$

The **ZP** (zero profits for lenders) curve is now

$$\pi(I)P(Z) + \left[1-\pi(I)\right]E_2 = (1+r)D = (1+r)(I - Y_1)$$

or

$$P(Z) - E_2 = \frac{(1+r)[I - Y_1 - E_2/(1+r)]}{\pi(I)}.$$

Thus the **IC** and **ZP** curves are analogous to those of the text, except that $P(Z)$ is replaced by $P(Z) - E_2$ and Y_1 is replaced by presented discounted collateralizable income $Y_1 + E_2/(1+r)$. Here, investment I responds to $Y_1 + E_2/(1+r)$ in the same way as to changes in Y_1.

(b) Now assume that the government has a debt outstanding to foreigners of D^{G} per capita, payable in the second period. The government, of course, can only impose second-period taxes on *successful* entrepreneurs. The government's budget constraint is given by

$$\pi(I)\tau = D^{\text{G}},$$

where τ is the tax on successful entrepreneurs. (Note that since the returns on various individual projects are independent, there is no uncertainty in the aggregate.) With this tax the **IC** curve becomes

$$\pi'(I)\{Z - [P(Z) + \tau]\} = 1 + r$$

or

$$P(Z) + \tau = Z - \frac{1+r}{\pi'(I)},$$

while the **ZP** curve remains the same as eq. (49) in Chapter 6. However, adding τ to both sides of the **ZP** curve and making use of the government budget constraint implies that the **ZP** can be written as

$$P(Z) + \tau = \frac{(1+r)[I - Y_1 + D^{\text{G}}/(1+r)]}{\pi(I)}.$$

A rise in D^{G} therefore has an effect exactly analogous to a fall in E_2 in part a. The overhang of government debt discourages investment.

5. First, note that in the decentralized economy, the presence of savers has no effect since the entrepreneurs already face an infinitely elastic supply of world savings. One can then show that a planner who has no information advantage over the private sector cannot achieve a Pareto improvement. One way to approach the problem is to have the planner maximize the utility of the representative entrepreneur subject to the constraint that savers earn at least the world market rate of return, that is, that

$$\pi(I)P(Z) + [1 - \pi(I)]\,P(0) \geq (1 + r)D.$$

This problem can be solved as in the text, in which case the same algorithm shows that the above constraint is binding. (If any multiplier in the problem is strictly positive, they all must be.)

To obtain a more intuitive understanding of the preceding argument, let us consider the tax redistribution scheme suggested in the problem. In the first-period, the government places a tax on savers of τ_1, and gives the proceeds to entrepreneurs. (Obviously, other things equal, this will raise total output though savers will be worse off.) Then, in the second period, the government issues a tax on (successful) entrepreneurs of τ_2, and uses the proceeds to pay back savers. If savers are to be made no worse off, then we must have

$$\tau_2\pi(I) \geq (1 + r)\tau_1.$$

We assume that there are an equal number of investors and savers. In the presence of transfers, the entrepreneur maximizes

$$\mathrm{E}C_2 = \pi(I)\,[Z - P(Z) - \tau_2] - [1 - \pi(I)]\,[P(0)] + (1 + r)\,(Y_1 + \tau_1 + D - I)\,.$$

The incentive compatibility constraint is now

$$\pi'(I)\,\{Z - [P(Z) + \tau_2 - P(0)]\} = 1 + r,$$

which, with $P(0) = 0$ (an unsuccessful entrepreneur does not pay the tax), reduces to

$$P(Z) + \tau_2 = Z - \frac{1 + r}{\pi'(I)}. \qquad (9)$$

Since entrepreneurs now borrow $D = I - Y_1 - \tau_1$, the zero-profit condition is

$$(1+r)\left(I - Y_1 - \tau_1\right) = \pi(I)P(Z) + \left[1 - \pi(I)\right]P(0).$$

If we substitute the condition $\tau_2\pi(I) = (1+r)\tau_1$ (which should hold so as not to make the savers worse off), the zero-profit condition can be written as

$$P(Z) + \tau_2 = \frac{(1+r)(I - Y_1)}{\pi(I)}. \tag{10}$$

It follows clearly from equations (9) and (10) that the **IC** and **ZP** curves are unchanged, except that $P(Z) + \tau_2$ has replaced $P(Z)$ on the vertical axis in figure 6.11 of the text. Each entrepreneur borrows τ_1 less, and the successful ones pay $(1+r)\tau_1/\pi(I) = \tau_2$ less to creditors. The intervention therefore has no effect on investment. The logic here is the same as in exercise 4, part b. Each entrepreneur's first-period income rises by τ_1. But the government must raise second-period taxes on successful entrepreneurs by an amount sufficient to raise revenue $(1+r)\tau_1$. This overhang effect exactly offsets the gain from the first-period subsidy. In general, there is no scope for Pareto-improving government intervention here despite the credit market imperfection.

6. (a) Assuming there is no debt forgiveness, the country's maximization problem can be written as

$$\max_I \left\{\log C_1 + \beta \log C_2\right\}$$

subject to

$$C_1 = Y_1 - I,$$

$$C_2 = (1-\eta)I^\alpha.$$

(By definition, the "inherited" debt specified in this exercise yields no benefits in period 1.) The maximization problem can be rewritten as

$$\max_I \left\{\log\left(Y_1 - I\right) + \beta \log\left[(1-\eta)I^\alpha\right]\right\}.$$

The first-order condition with respect to I is

$$\frac{1}{Y_1 - I} = \beta \frac{1}{(1-\eta)I^{\alpha}}(1-\eta)\alpha I^{\alpha-1} = \frac{\alpha\beta}{I},$$

giving an investment level of

$$I^{\mathrm{D}} = \frac{\alpha\beta Y_1}{1+\alpha\beta}$$

when default is planned. The country's repayment is then the (forcibly extracted) amount

$$\mathcal{R}^{\mathrm{D}} = \eta\left(\frac{\alpha\beta Y_1}{1+\alpha\beta}\right)^{\alpha}.$$

The equilibrium is at point **A** in figure 6.2 (cf. figure 6.3 on p. 382 in the chapter). Note that the parameter η, which effectively determines the productivity of investment, does not affect the optimal level of investment. This is the result of our log utility specification which implies that income and substitution effects exactly cancel and thus that the "tax" rate η does not affect savings.

(b) Now assume that creditors agree to write down the country's very large debt to $\mathcal{R}^{\mathrm{D}}$. That is, once the country has paid $\mathcal{R}^{\mathrm{D}}$, it does not owe anything further. Why does this affect the country's maximization problem? Intuitively, once the country pays back its debt, the marginal "tax" on investment drops to zero.

With the debt written down to $\mathcal{R}^{\mathrm{D}} = \eta\left(\frac{\alpha\beta Y_1}{1+\alpha\beta}\right)^{\alpha}$, the period 2 budget constraint becomes

$$C_2 = \max\left\{(1-\eta)Y_2,\ Y_2 - \eta\left(\frac{\alpha\beta Y_1}{1+\alpha\beta}\right)^{\alpha}\right\}.$$

It is easiest to proceed by assuming that the country is going to pay back in full; it is easy to show that after forgiveness to $\mathcal{R}^{\mathrm{D}}$, this strategy dominates any strategy involving default. The country's maximization problem now becomes

$$\max_{C_1}\left\{\log C_1 + \beta\log\left[(Y_1 - C_1)^{\alpha} - \mathcal{R}^{\mathrm{D}}\right]\right\}$$

and the first-order condition becomes

$$\frac{1}{C_1} = \frac{\beta}{C_2}\alpha(Y_1 - C_1)^{\alpha-1} = \frac{\alpha\beta}{Y_1 - C_1 - \mathcal{R}^{\text{D}}(Y_1 - C_1)^{1-\alpha}} > \frac{\alpha\beta}{Y_1 - C_1},$$

from which it immediately follows that C_1 must be lower with debt forgiveness than its level $C_1 = Y_1/(1+\alpha\beta)$ in part a. Correspondingly, investment must be higher. (Intuitively, in comparison to part a, two things happen: η falls to zero, which leaves investment unchanged under log preferences, but a second-period lump-sum levy of $\mathcal{R}^{\text{D}}$ is imposed, reducing consumption and raising investment.) The country must also be better off, since with debt forgiveness to $\mathcal{R}^{\text{D}}$, it always has the option of replicating its allocation in the no-forgiveness equilibrium. We can also see this graphically in figure 6.3.

Figure 6.3 is similar to the graph on p. 382 in the text. The line GDP represents the production possibilities frontier for the small country. GNP^{D} represents the budget constraint for the country in the case where the debt is not reduced and repayment of ηY_2 is enforced. As discussed in part a of this problem the equilibrium for the economy is at **A**. GNP^{N} represents the budget constraint for the economy when the debt is reduced to $\mathcal{R}^{\text{D}}$ and the country does not default. Clearly GNP^{N} intersects GNP^{D} at point **A** and has a steeper slope at that point. There is therefore a point to the left of **A** on the GNP^{N} curve (**A′**) that the small country strictly prefers to point **A**. This ranking can be seen from the tangencies of the indifference curves.

(c) In figure 6.3 we see that by writing the debt down to the level $\mathcal{R}^{\text{D}}$ defined in part b, lenders can raise borrower investment and welfare while averting default and leaving their own repayment unchanged at $\eta\,(I^{\text{D}})^{\alpha}$. At point **A′** in figure 6.3, however, the borrower *strictly* prefers repayment to default. Thus, lenders could forgive slightly less of the country's debt, shifting GNP^{N} downward and inducing the country to invest and repay even more than the amounts shown in figure 6.3. The country's optimal investment given a binding enforcement constraint is $I^{\text{D}} = \alpha\beta Y_1/(1+\alpha\beta)$, which yields a utility level under default of

$$U^{\mathrm{D}} = \log\left(\frac{Y_1}{1+\alpha\beta}\right) + \beta \log\left[(1-\eta)\left(\frac{\alpha\beta Y_1}{1+\alpha\beta}\right)^{\alpha}\right].$$

Selfish creditors would write the debt down by the smallest amount needed to dissuade the country from defaulting (and thereby obtaining utility U^{D}). The problem for the creditors thus can be written as

$$\max \mathcal{R}$$

subject to

$$0 = \frac{I}{Y_1 - I} - \frac{\alpha\beta I^{\alpha}}{I^{\alpha} - \mathcal{R}} \tag{11}$$

and

$$U^{\mathrm{D}} \leq \log(Y_1 - I) + \beta \log\left(I^{\alpha} - \mathcal{R}\right),$$

where condition (11) states that for a given level of remaining debt $\mathcal{R}$ after forgiveness, investment is chosen optimally by the indebted country, given that it repays $\mathcal{R}$ in full. The optimal level of debt after forgiveness, $\mathcal{R}^*$, is represented in figure 6.4. $\mathrm{GNP}^{\mathrm{R}}$ corresponds to the nondefault budget constraint when $\mathcal{R} = \mathcal{R}^*$. (It is simply the curve $\mathrm{GNP}^{\mathrm{N}}$ corresponding to the repayment-maximizing level of debt forgiveness.) Clearly, the optimal $\mathcal{R}$ is greater than $\mathcal{R}^{\mathrm{D}}$. The optimal strategy for creditors is to lower debt to the point where the debtor is just indifferent between its optimal strategies under repayment and under default. The optimal consumption point for the country is **B** (as drawn in figure 6.4). Investment I^{R} is plainly greater than I^{D}.