Foundations of International Macroeconomics[1]

Workbook[2]

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Chapter 3 Solutions

1. (a) Because $r = 0$, an individual's *desired* consumption when young would be

$$c^{\mathrm{Y}} = \frac{1}{3}\left[y^{\mathrm{Y}} + (1+e)\, y^{\mathrm{Y}}\right] = \frac{1}{3}(2+e)\, y^{\mathrm{Y}}$$

if unrestricted borrowing were possible. In general

$$c^{\mathrm{Y}} = \min\left\{y^{\mathrm{Y}}, \frac{1}{3}(2+e)\, y^{\mathrm{Y}}\right\}.$$

Obviously the borrowing constraint will bind only when $e > 1$. Since $\beta = 1$, $c^{\mathrm{O}} = c^{\mathrm{M}} = c^{\mathrm{Y}}$ when the borrowing constraint of the young doesn't bind. If it does (that is, if $e > 1$), then

$$c^{\mathrm{O}} = c^{\mathrm{M}} = \frac{1}{2}(1+e)\, y^{\mathrm{Y}}.$$

The saving of a young person is

$$s^{\mathrm{Y}} = \max\left\{0, y^{\mathrm{Y}} - \frac{1}{3}(2+e)\, y^{\mathrm{Y}}\right\} = \max\left\{0, \frac{1-e}{3} y^{\mathrm{Y}}\right\}.$$

That of a middle-aged person is

$$s^{\mathrm{M}} = (1+e)\, y^{\mathrm{Y}} - \frac{(1+e)\, y^{\mathrm{Y}} + s^{\mathrm{Y}}}{2} = \frac{(1+e)\, y^{\mathrm{Y}} - s^{\mathrm{Y}}}{2},$$

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and that of an old person is

$$s^{\mathrm{O}} = -(s^{\mathrm{Y}} + s^{\mathrm{M}}) = -\left[\frac{(1+e)\,y^{\mathrm{Y}} + s^{\mathrm{Y}}}{2}\right].$$

(b) Now $1+g$ is the gross growth rate of a young-person's output. Aggregate saving out of total output is

$$\frac{s_t^{\mathrm{Y}} + s_t^{\mathrm{M}} + s_t^{\mathrm{O}}}{y_t^{\mathrm{Y}} + y_t^{\mathrm{O}}},$$

or

$$\begin{array}{l}\text{Aggregate} \\ \text{saving rate}\end{array} = \frac{\max\left\{0, \frac{1-e}{3} y_t^{\mathrm{Y}}\right\} + \frac{(1+e)\,y_{t-1}^{\mathrm{Y}} - s_{t-1}^{\mathrm{Y}}}{2} - \left[\frac{(1+e)\,y_{t-2}^{\mathrm{Y}} + s_{t-2}^{\mathrm{Y}}}{2}\right]}{[(1+g)+(1+e)]\,y_{t-1}^{\mathrm{Y}}}. \tag{1}$$

Let us first assume that $e \leq 1$, so that the young do not wish to borrow a positive amount. In that case, eq. (1) becomes

$$\begin{aligned}\begin{array}{l}\text{Aggregate} \\ \text{saving rate}\end{array} &= \frac{\frac{1-e}{3}(1+g)y_{t-1}^{\mathrm{Y}} + \frac{1+2e}{3} y_{t-1}^{\mathrm{Y}} - \frac{2+e}{3(1+g)} y_{t-1}^{\mathrm{Y}}}{[(1+g)+(1+e)]\,y_{t-1}^{\mathrm{Y}}} \\ &= \frac{(1-e)(1+g)^2 + (1+2e)(1+g) - (2+e)}{3\,[(1+g)^2 + (1+g)(1+e)]}. \end{aligned} \tag{2}$$

Borrowing by the young becomes an issue only if $e > 1$. If the young cannot borrow $s^{\mathrm{Y}} = 0$ and aggregate saving is computed accordingly, taking account of the fact s^{M} and s^{O} are *not* the same as when the young can borrow freely. In the borrowing-constrained case, eq. (2) is replaced by

$$\begin{aligned}\begin{array}{l}\text{Aggregate} \\ \text{saving rate}\end{array} &= \frac{s_t^{\mathrm{M}} + s_t^{\mathrm{O}}}{y_t^{\mathrm{Y}} + y_t^{\mathrm{O}}} = \frac{\frac{(1+e)\,y_{t-1}^{\mathrm{Y}}}{2} - \frac{(1+e)\,y_{t-2}^{\mathrm{Y}}}{2}}{[(1+g)+(1+e)]\,y_{t-1}^{\mathrm{Y}}} \\ &= \frac{\frac{(1+e)}{2} - \frac{(1+e)}{2(1+g)}}{(1+g)+(1+e)} \\ &= \frac{g(1+e)}{2\,[(1+g)^2 + (1+g)(1+e)]}. \end{aligned} \tag{3}$$

(c) Take the derivative with respect to e of the numerator of eq. (2). It is

$$-(1+g)^2 + 2(1+g) - 1 = -g^2 < 0.$$

Because, in addition, the denominator of (2) rises when e rises, steeper income growth between youth and middle age depresses saving. Intuitively, the young and old save less and the middle-aged save more, but with positive overall economic growth ($g > 0$), it is the effects on the young and old that dominate. When the young can't borrow and $e > 1$, the derivative is computed from eq. (3) and is proportional to $2(1+g)g > 0$. The preceding effect is reversed: the positive effect of a higher e on middle-age saving dominates.

(d) Observe that

$$\begin{aligned} s_t^{\mathrm{Y}} &= \frac{2}{3} y_t^{\mathrm{Y}} - \frac{1}{3} y_{t+1}^{\mathrm{M}}, \\ s_t^{\mathrm{M}} &= \frac{2}{3} y_t^{\mathrm{M}} - \frac{1}{3} y_{t-1}^{\mathrm{Y}}, \\ s_t^{\mathrm{O}} &= -\frac{1}{3}\left(y_{t-2}^{\mathrm{Y}} + y_{t-1}^{\mathrm{M}}\right). \end{aligned}$$

Thus, in the first case described in this section of the exercise, the aggregate saving rate is

$$\begin{aligned} \begin{array}{l}\text{Aggregate} \\ \text{saving rate}\end{array} &= \frac{\left[\frac{2}{3}(1+g)^2 y_{t-2}^{\mathrm{Y}} - \frac{1}{3} y^{\mathrm{M}}\right] + \left[\frac{2}{3} y^{\mathrm{M}} - \frac{1}{3}(1+g) y_{t-2}^{\mathrm{Y}}\right] - \frac{1}{3}\left(y_{t-2}^{\mathrm{Y}} + y^{\mathrm{M}}\right)}{(1+g)^2 y_{t-2}^{\mathrm{Y}} + y^{\mathrm{M}}} \\ &= \frac{\left(\frac{2}{3} g^2 + g\right) y_{t-2}^{\mathrm{Y}}}{(1+g)^2 y_{t-2}^{\mathrm{Y}} + y^{\mathrm{M}}}. \end{aligned}$$

Taking the derivative above with respect to g, we find it is proportional to

$$\left(1 + \frac{4}{3} g\right) y^{\mathrm{M}} + \left(1 + \frac{4}{3} g + \frac{1}{3} g^2\right) y_{t-2}^{\mathrm{Y}} > 0.$$

Thus countries with higher output growth rates for young workers will have higher saving rates (all else equal). In the second case, in which growth is concentrated on the middle-aged,

$$\begin{aligned}\begin{array}{l}\text{Aggregate}\\ \text{saving rate}\end{array} &= \frac{\left[\frac{2}{3}y^{\mathrm{Y}}-\frac{1}{3}(1+g)^2 y^{\mathrm{M}}_{t-1}\right]+\left[\frac{2}{3}(1+g)y^{\mathrm{M}}_{t-1}-\frac{1}{3}y^{\mathrm{Y}}\right]-\frac{1}{3}\left(y^{\mathrm{Y}}+y^{\mathrm{M}}_{t-1}\right)}{y^{\mathrm{Y}}+(1+g)y^{\mathrm{M}}_{t-1}}\\ &= \frac{-\frac{1}{3}g^2 y^{\mathrm{M}}_{t-1}}{y^{\mathrm{Y}}+(1+g)y^{\mathrm{M}}_{t-1}}.\end{aligned}$$

It is easy to see that the derivative with respect to g of the last expression is negative. Growth concentrated in middle-aged workers lowers the national saving rate if the young can borrow against future earnings.

2. (a) With log utility,

$$c^{\mathrm{Y}}_t = \frac{w_t}{1+\beta}, \qquad c^{\mathrm{O}}_{t+1} = \frac{(1+r)\beta w_t}{1+\beta}.$$

The indirect utility function of a young agent is then

$$U = (1+\beta)\log(w) + \beta\log(1+r),$$

apart from an irrelevant additive constant. Differentiation yields

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \left(\frac{1+\beta}{w}\right)\frac{\mathrm{d}w}{\mathrm{d}r} + \frac{\beta}{1+r}.$$

For an infinitesimal change in r, using the envelope theorem we have that $\mathrm{d}w/\mathrm{d}r = -k$, so that

$$\frac{\mathrm{d}U}{\mathrm{d}r} = -\left(\frac{1+\beta}{w}\right)k + \frac{\beta}{1+r}. \tag{4}$$

A representative young agent saves $s^{\mathrm{Y}}_t = w_t - w_t/(1+\beta) = \beta w_t/(1+\beta)$ on date t. Thus, starting from a situation where the world and autarky interest rates are equal, we have

$$\frac{K_{t+1}}{N_{t+1}} = \frac{N_t s^{\mathrm{Y}}_t}{N_{t+1}},$$

which can be rewritten in per capita terms as

$$k_{t+1} = \frac{s_t^{\mathrm{Y}}}{(1+n)} = \frac{\beta w_t}{(1+\beta)(1+n)}.$$

Substituting for k in eq. (4) yields

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \frac{-\beta}{1+n} + \frac{\beta}{1+r}.$$

Hence $\mathrm{d}U/\mathrm{d}r > 0$—and the current young and all future generations benefit from a rise in r on date t—if and only if $r < n$, as the exercise assumes. [Because saving per worker is constant after the change and $k = s^{\mathrm{Y}}/(1+n)$, we can express the lifetime income change for a new generation as $-k\mathrm{d}r + \frac{1+n}{1+r}k\mathrm{d}r > 0$.] The date t old benefit because of a higher return on their previous saving.

(b) Opening the economy to trade will bring the domestic autarky interest rate down to the world level. Suppose the world rate r is infinitesimally below r^{A}. Then the change makes all generations worse off if $r^{\mathrm{A}} < n$. The problem is that we are removing one distortion, trade barriers, while leaving dynamic inefficiency uncorrected. Further, opening to trade exacerbates dynamic inefficiency because $r < r^{\mathrm{A}} < n$. (The result is *not* true in general for non-infinitesimal gaps between the autarky and world interest rates. Closed-economy intuition may be misleading in this case, because in an open economy, as r falls the economy does not have to save more to maintain an ever-higher capital-labor ratio. It borrows for that purpose instead, and in the dynamically inefficient case, higher steady-state per capita foreign debt implies higher—not lower—steady state per capita consumption.)

3. [There are two typos in the statement of this exercise. One line up from the bottom of p. 195, $u(c_t)$ should be $u(c_s)$. On p. 196 in part c, the term $(y_t^v - \tau_t^v)$ should be $(y_s^v - \tau_s^v)$.]

(a) On any date t, all the members of the generation born on t still are alive, and contribute 1 to population. Of those born on $t-1$, $\varphi \cdot 1 = \varphi$ remain on date t. Only $\varphi - (1-\varphi)\varphi = \varphi^2$ of those born on date $t-2$ are still around on date t. And so on. Total population on any date therefore is

$$1 + \varphi + \varphi^2 + \varphi^3 + ... = 1/(1-\varphi).$$

(b) The insurance industry pays a gross return of $(1+r)/\varphi$ to those who actually survive the period, but nothing to those who don't. Only a fraction φ of the existing population makes it from date t into date $t+1$. Thus the industry's gross payout on the assets B (positive or negative) that it holds is

$$\varphi \frac{1+r}{\varphi} B = (1+r)B,$$

which exactly equals its earnings. Hence profits are zero.

(c) Given the effective interest rate individuals face, the asset-accumulation identity is

$$b_{t+1}^{\mathrm{P},v} = \frac{1+r}{\varphi} b_t^{\mathrm{P},v} + y_t^v - \tau_t^v - c_t^v, \tag{5}$$

or, in terms of the lag operator,

$$(1 - \frac{\varphi}{1+r} L^{-1}) \left(\frac{1+r}{\varphi}\right) b_t^{\mathrm{P},v} = c_t^v - (y_t^v - \tau_t^v).$$

Invert the left-hand side lag polynomial above and impose the condition $\lim_{T\to\infty} \left(\frac{\varphi}{1+r}\right)^T b_{t+T+1}^{\mathrm{P},v} = 0$. The result is the intertemporal budget constraint

$$\left(\frac{1+r}{\varphi}\right) b_t^{\mathrm{P},v} = \sum_{s=t}^{\infty} \left(\frac{\varphi}{1+r}\right)^{s-t} \left[c_s^v - (y_s^v - \tau_s^v)\right].$$

(d) We know that for log consumption the individual's optimum plan is

$$c_t^v = (1-\varphi\beta) \left[\left(\frac{1+r}{\varphi}\right) b_t^{\mathrm{P},v} + \sum_{s=t}^{\infty} \left(\frac{\varphi}{1+r}\right)^{s-t} (y_s^v - \tau_s^v) \right] \tag{6}$$

(because the *effective* subjective discount factor is $\varphi\beta$). The aggregated version of eq. (5) is slightly intricate to derive, but we use the parenthetical hint at the end of the question for this part. Recall that $b_t^{\text{P},v}$ is the end-of-$(t-1)$ assets of someone from vintage v. Thus for the economy as a whole,

$$\begin{aligned} B_t^{\text{P}} &= 1 \cdot b_t^{\text{P},t-1} + \varphi \cdot b_t^{\text{P},t-2} + \varphi^2 \cdot b_t^{\text{P},t-3} + ..., \\ B_{t+1}^{\text{P}} &= 1 \cdot b_{t+1}^{\text{P},t} + \varphi \cdot b_{t+1}^{\text{P},t-1} + \varphi^2 \cdot b_{t+1}^{\text{P},t-2} + ..., \end{aligned}$$

while, in contrast,

$$Y_t = 1 \cdot y_t^t + \varphi \cdot y_t^{t-1} + \varphi^2 \cdot y_t^{t-2} + ...,$$

etc. Being careful about time subscripts, we therefore may aggregate eq. (6) as

$$\begin{aligned} C_t &= (1-\varphi\beta) \left\{ \begin{array}{c} \left(\frac{1+r}{\varphi}\right) \left[1 \cdot b_t^{\text{P},t} + \varphi \cdot b_t^{\text{P},t-1} + \varphi^2 \cdot b_t^{\text{P},t-2} + ...\right] \\ + \sum_{s=t}^{\infty} \left(\frac{\varphi}{1+r}\right)^{s-t} (Y_s - T_s) \end{array} \right\} \\ &= (1-\varphi\beta) \left\{ \begin{array}{c} \left(\frac{1+r}{\varphi}\right) \varphi \left[1 \cdot b_t^{\text{P},t-1} + \varphi \cdot b_t^{\text{P},t-2} + ...\right] \\ + \sum_{s=t}^{\infty} \left(\frac{\varphi}{1+r}\right)^{s-t} (Y_s - T_s) \end{array} \right\} \\ &= (1-\varphi\beta) \left[(1+r) B_t^{\text{P}} + \sum_{s=t}^{\infty} \left(\frac{\varphi}{1+r}\right)^{s-t} (Y_s - T_s) \right], \end{aligned} \tag{7}$$

where the fact that $b_t^{\text{P},t} = 0$ (the newly born have no financial wealth at the start of the period they are born) has been used. Similarly, we aggregate eq. (5) as

$$\begin{aligned} B_{t+1}^{\text{P}} &= \left[1 \cdot b_{t+1}^{\text{P},t} + \varphi \cdot b_{t+1}^{\text{P},t-1} + \varphi^2 \cdot b_{t+1}^{\text{P},t-2} + ...\right] \\ &= \frac{(1+r)}{\varphi} \left[1 \cdot b_t^{\text{P},t} + \varphi \cdot b_t^{\text{P},t-1} + \varphi^2 \cdot b_t^{\text{P},t-2} + ...\right] + Y_t - T_t - C_t \\ &= \frac{(1+r)}{\varphi} \varphi \left[1 \cdot b_t^{\text{P},t-1} + \varphi \cdot b_t^{\text{P},t-2} + ...\right] + Y_t - T_t - C_t \\ &= (1+r) B_t^{\text{P}} + Y_t - T_t - C_t. \end{aligned}$$

The intuition for this relation (which is the usual one) is that the economy as a whole is earning the interest rate r on its net foreign assets.

(e) If Y and T are constants, eq. (7) can be substituted into the last aggregate equation to yield

$$\begin{aligned} B_{t+1}^{\mathrm{P}} &= (1+r)B_t^{\mathrm{P}} + Y - T - (1-\varphi\beta)\left[(1+r)\,B_t^{\mathrm{P}} + \frac{(1+r)\,(Y-T)}{1+r-\varphi}\right] \\ &= (1+r)\varphi\beta B_t^{\mathrm{P}} + \varphi\left[\frac{(1+r)\beta-1}{1+r-\varphi}\right](Y-T). \end{aligned}$$

The dynamics are qualitatively the same those shown in figure 3.9, assuming that the system is stable [meaning that $1 > (1+r)\beta\varphi$]. The slope of the flatter of the two diagonal lines is now $(1+r)\beta\varphi$, however, and the line's vertical intercept is $\frac{(1+r)\beta-1}{1+r-\varphi}(Y-T)$.

(f) We can solve for steady state private assets using the equation in part e. We get, setting $B_{t+1}^{\mathrm{P}} = B_t^{\mathrm{P}} = \bar{B}^{\mathrm{P}}$,

$$\bar{B}^{\mathrm{P}} = \frac{(1+r)\beta-1}{(1+r-\varphi)\left[1-(1+r)\varphi\beta\right]}\varphi(Y-T),$$

and, for steady state aggregate consumption,

$$\bar{C} = \left(\frac{1-\varphi}{1+r-\varphi}\right)\left(\frac{1-\varphi\beta}{1-(1+r)\varphi\beta}\right)(1+r)(Y-T).$$

By substituting $T = rD$ above, we can see the effects of a steady state public debt of D. Clearly a higher D depresses steady-state consumption, from the last equation, assuming the dynamic stability condition (which is needed for positive steady state consumption). Since output is exogenous, this must mean that steady-state total net foreign assets—the sum of government and private net foreign assets— are lower. That is, private net foreign assets rise by less than government debt. Note that in the Ricardian case ($\varphi = 1$), we have, in contrast,

$$\bar{B}^{\mathrm{P}} = -\frac{Y}{r} + D,$$

so that a rise in D raises steady-state net private foreign assets one-for-one.

4. In the case of a dynamically inefficient world economy with $r < n$ (see section 3.6.4), the introduction of a public debt financed entirely by taxes on the young can raise the welfare of all generations, and in both countries too! If the two countries initially finance their capital stocks entirely out of their own savings, with no net asset trade, then the introduction of a small public debt in Home reduces the world's dynamic inefficiency without any international redistribution effects. As in exercise 2, part a, above, everyone in the world benefits from the world interest rate rise, and Home's young pay lower taxes to their government as explained in section 3.6.4. Net international private lending changes the analysis, however, by introducing redistribution effects due to intertemporal terms of trade changes. When Home is initially a net creditor of Foreign, for example, the interest rate rise creates a further terms-of-trade benefit for Home, but at Foreign's expense. So Foreign's current and future generations may be net losers despite the gains in Home. See section 3.5 on such international redistribution effects.

5. Intuitively, we know that $(1+r)nd_t/r$ (where $n < r$) is the contribution of the existing public debt to the net wealth of vintages alive on date t. Any further debt issues $d_{s+1} - d_s$ will raise net wealth on date s by the amount

$$(1+r)n(d_{s+1} - d_s)/r.$$

This explains the consumption function you are asked to derive. To derive the expression formally, rewrite the government finance constraint as

$$\begin{aligned} \tau_t &= (1+r)d_t - (1+n)d_{t+1} + g_t \\ &= \frac{1+r}{r}(r-n)\left(d_t - \frac{1}{1+r}d_{t+1}\right) - \frac{1+r}{r}n(d_{t+1} - d_t) + g_t. \end{aligned}$$

(Don't be depressed if this last equality isn't immediately obvious. But do be sure to combine terms to check its validity.) Let L^{-1} denote the lead operator, as usual (see supplement C to Chapter 2), and write the last equality as

$$\tau_t = \frac{1+r}{r}(r-n)\left[1 - (1+r)^{-1}L^{-1}\right]d_t - \frac{1+r}{r}n(d_{t+1} - d_t) + g_t. \quad (8)$$

Observe next that the present discounted value of per capita current and future taxes can be expressed as

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tau_s=\left[1-(1+r)^{-1}L^{-1}\right]^{-1}\tau_t.$$

Substituting for taxes using formula (8), we find that:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tau_s=\frac{1+r}{r}(r-n)d_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[g_s-\frac{1+r}{r}n\left(d_{s+1}-d_s\right)\right].$$

Substitution of the above equation for the present value of aggregate per capita taxes in eq. (66) of Chapter 3 yields the desired result. One also has to make use of the identity $b=b^{\mathrm{p}}-d$ (i.e., the economy's total net foreign assets are the sum of public net assets and private net assets).

6. The government is not indifferent as to the path of distorting taxes, because different tax paths of equal present value may inflict different total distortion costs on the economy. To see this, solve the government's planning problem. The government picks the paths of private consumption and taxes to maximize

$$U_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}u(C_s)$$

subject to the consumer's rule for intertemporal consumption smoothing,

$$u'(C_s)=u'(C_{s+1}),$$

and subject to the private-sector budget constraint,

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s-\frac{aT_s^2}{2}-T_s-C_s\right)=0,$$

and the government constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(T_s-G_s)=0.$$

(For simplicity a zero date t public debt is assumed.) The private Euler equation implies that desired consumption is flat at some level $\bar{C}$. Thus, we can write the Lagrangian for the government's problem as

$$\mathcal{L}_t = \frac{1+r}{r}\left[u(\bar{C}) - \lambda\bar{C}\right] - \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\lambda\left(Y_s - \frac{aT_s^2}{2} - T_s\right) + \eta\left(T_s - G_s\right)\right]$$

where λ is the Lagrange multiplier on the private consumption constraint and η that on the government budget constraint. The first order conditions, found by differentiating with respect to $\bar{C}$ and T_s ($\forall s \geq t$) are:

$$u'(\bar{C}) = \lambda, \quad \eta - u'(\bar{C}) = u'(\bar{C})aT_s.$$

The second of these conditions states that at an optimum, the shadow value of government resources, η, exceeds their private value, $u'(\bar{C})$, by an amount equal to the marginal deadweight loss inflicted by the tax (measured in utility). Because both η and $u'(\bar{C})$ are constant over time, however, taxes are constant over time as well. In other words, the government finds it optimal to smooth taxes, just as consumers smooth consumption. The constant level of tax is

$$\bar{T} = \frac{\eta - u'(\bar{C})}{au'(\bar{C})}.$$

One solves for η using the government budget constraint,

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\frac{\eta - u'(\bar{C})}{au'(\bar{C})} - G_s\right] = 0,$$

to find

$$\eta = u'(\bar{C}) + \frac{r}{1+r}u'(\bar{C})\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} aG_s.$$

[Interpretation: the government shadow value of revenue η exceeds $u'(\bar{C})$ by a weighted average of current and future marginal consumption costs due to the exogenous stream of public expenditures.] Since there is an optimal level of taxes, there is also an optimal public deficit on each date, in contrast to Ricardian equivalence models. When G is unusually high, the government will run a deficit rather than raising taxes, and when G is unusually low it will run a surplus.