Foundations of International Macroeconomics[1]

Workbook[2]

Maurice Obstfeld, Kenneth Rogoff, and Gita Gopinath

Chapter 1 Solutions

1. (a) The intertemporal budget constraint can be expressed as

$$C_2 = (1+r)(Y_1 - C_1) + Y_2.$$

Substitute this expression for C_2 into lifetime utility $U(C_1, C_2)$ to obtain

$$U = U\left[C_1, (1+r)(Y_1 - C_1) + Y_2\right]. \tag{1}$$

Taking the total derivative of U with respect to C_1 and equating it to zero, one gets the following Euler equation for optimal consumption:

$$\frac{\partial U(C_1, C_2)}{\partial C_1} = (1+r)\frac{\partial U(C_1, C_2)}{\partial C_2}.$$

(b) Total differentiation of expression (1) with respect to r gives

$$\begin{aligned}
\frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{\partial U(C_1, C_2)}{\partial C_1}\frac{\mathrm{d}C_1}{\mathrm{d}r} + \frac{\partial U(C_1, C_2)}{\partial C_2}\left[(Y_1 - C_1) - (1+r)\frac{\mathrm{d}C_1}{\mathrm{d}r}\right] \\
&= \left[\frac{\partial U(C_1, C_2)}{\partial C_1} - (1+r)\frac{\partial U(C_1, C_2)}{\partial C_2}\right]\frac{\mathrm{d}C_1}{\mathrm{d}r} + \frac{\partial U(C_1, C_2)}{\partial C_2}(Y_1 - C_1) \\
&= \frac{\partial U(C_1, C_2)}{\partial C_2}(Y_1 - C_1), \qquad (2)
\end{aligned}$$

where the Euler equation from part a gives the last equality. (Notice another instance of the envelope theorem.)

(c) Recall the relationship derived in part b,

$$\frac{\mathrm{d}U\left(C_1,C_2\right)}{\mathrm{d}r}=\frac{\partial U(C_1,C_2)}{\partial C_2}\left(Y_1-C_1\right).$$

Because the marginal utility of consumption is positive, the country will benefit from an increase in r if and only if $Y_1-C_1>0$, that is, if it is a net lender in period 1. In that case, the increase in r corresponds to an improvement in the country's intertemporal terms of trade.

(d) Write expression (1) as

$$U=U\left[C_1,(1+r)\left(W_1-C_1\right)\right].$$

Differentiation with respect to W_1 gives

$$\frac{\mathrm{d}U}{\mathrm{d}W_1}=(1+r)\frac{\partial U(C_1,C_2)}{\partial C_2}.$$

(By the envelope theorem, the two terms in $\mathrm{d}C_1/\mathrm{d}W_1$ cancel each other.) If $\mathrm{d}W_1=\widehat{r}(Y_1-C_1)$, where $\widehat{r}\equiv\mathrm{d}r/(1+r)$, then by the Euler equation,

$$\mathrm{d}U=(1+r)\frac{\partial U(C_1,C_2)}{\partial C_2}\widehat{r}(Y_1-C_1)=\frac{\partial U(C_1,C_2)}{\partial C_1}\widehat{r}\left(Y_1-C_1\right).$$

But eq. (2) in part b implies that the welfare effect of a percentage gross interest rate change $\widehat{r}$ is the same,

$$\mathrm{d}U=\frac{\partial U(C_1,C_2)}{\partial C_2}(Y_1-C_1)\mathrm{d}r=\frac{\partial U(C_1,C_2)}{\partial C_1}\widehat{r}\left(Y_1-C_1\right).$$

Thus the interest-rate change alters lifetime wealth by the amount $\widehat{r}\left(Y_1-C_1\right).$ This is the usual type of formula for a terms-of-trade effect, as computed in static international trade theory.

2. (a) Substitution of the consumption Euler equation

$$C_2=\beta(1+r)C_1$$

into the intertemporal budget constraint gives

$$C_1(r) = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right).$$

(b) Home saving is derived as

$$S_1(r) = Y_1 - C_1(r) = \frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2. \tag{3}$$

A parallel expression gives Foreign saving.

(c) Global equilibrium requires that

$$S_1(r) + S_1^*(r) = 0.$$

Substituting into this relationship the expressions for saving from part b gives

$$1 + r = \frac{\dfrac{Y_2}{1+\beta} + \dfrac{Y_2^*}{1+\beta^*}}{\dfrac{\beta Y_1}{1+\beta} + \dfrac{\beta^* Y_1^*}{1+\beta^*}}.$$

(d) In autarky, $S_1(r^{\mathrm{A}}) = S_1^*(r^{\mathrm{A}*}) = 0$, so that

$$1 + r^{\mathrm{A}} = \frac{Y_2}{\beta Y_1},$$

$$1 + r^{\mathrm{A}*} = \frac{Y_2^*}{\beta^* Y_1^*}.$$

Using these solutions to eliminate Y_2 and Y_2^* from the expression for the equilibrium world interest rate in part c, one obtains

$$r = \frac{\beta(1+\beta^*)Y_1}{\beta(1+\beta^*)Y_1 + \beta^*(1+\beta)Y_1^*}r^{\mathrm{A}} + \frac{\beta^*(1+\beta)Y_1^*}{\beta(1+\beta^*)Y_1 + \beta^*(1+\beta)Y_1^*}r^{\mathrm{A}*}.$$

Because the world interest rate is a weighted average of the two autarky rates, it must lie between them.

(e) The expression for Home's current account is given by eq. (3) and can be written as

$$CA_1 = S_1 = \frac{Y_2}{1+\beta}\left(\frac{1}{1+r^{\mathrm{A}}} - \frac{1}{1+r}\right) = \frac{Y_2}{1+\beta}\left[\frac{r-r^{\mathrm{A}}}{(1+r^{\mathrm{A}})(1+r)}\right],$$

where the autarky interest rate formula of part d has been used to make the substitution $\beta Y_1 = Y_2/(1+r^{\mathrm{A}})$ in eq. (3). Plainly Home will run a date 1 current account surplus, $CA_1 > 0$, if and only if $r^{\mathrm{A}} < r$. An autarky interest rate above r implies $CA_1 < 0$.

(f) From the expression for the equilibrium world interest rate in part d, it is easy to see that an increase in Y_2^*/Y_1^* raises r by raising Foreign's autarky interest rate. Moreover, given that

$$\frac{\mathrm{d}U_1\left(C_1, C_2\right)}{\mathrm{d}r} = \frac{\partial U_1(C_1, C_2)}{\partial C_2}\left(Y_1 - C_1\right),$$

(see part b of question 1), the consumption Euler equation and the expression for $C_1(r)$ from part a imply.

$$\begin{aligned}
\frac{\mathrm{d}U_1}{\mathrm{d}r} &= \frac{1}{1+r}\left[\frac{1+\beta}{Y_1 + Y_2/(1+r)}\right]\left[\frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2\right] \\
&= \frac{1}{1+r}\left[\frac{(1+r)\beta Y_1 - Y_2}{(1+r)Y_1 + Y_2}\right] \\
&= \frac{\beta}{1+r}\left[\frac{(1+r) - \dfrac{Y_2}{\beta Y_1}}{(1+r) + \dfrac{Y_2}{Y_1}}\right] \\
&= \frac{\beta}{1+r}\left[\frac{r - r^{\mathrm{A}}}{(1+r) + \beta(1+r^{\mathrm{A}})}\right].
\end{aligned}$$

If the world interest rate exceeds Home's autarky rate, Home runs a date 1 current account surplus, as shown in part e. In that case a higher Foreign rate of output growth causes r to be higher, and Home's welfare is enhanced via the favorable intertemporal terms of trade effect. Home's welfare rises because the rise in Foreign growth widens the spread between autarky interest rates, increasing the gains from intertemporal trade. If Home were instead a date 1 borrower, a rise in Foreign output growth would reduce the gains from trade and also reduce Home welfare.

3. (a) From the equality of the interest rate r and the marginal product of capital

$$\alpha A_2 K_2^{\alpha-1} = r,$$

one obtains

$$K_2 = \left(\frac{\alpha A_2}{r}\right)^{\frac{1}{1-\alpha}}.$$

(b) From the identity $I_1 = K_2 - K_1$, substitution of the above expression for K_2 gives the function $I_1(r)$.

(c) Given that utility is logarithmic, consumption at time 1 is a fraction $1/1+\beta$ of lifetime wealth. From the intertemporal budget constraint

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 + \frac{Y_2}{1+r},$$

one readily gets $C_1(r)$.

(d) Given $I_1(r)$ from part b, $C_1(r)$ from part c, and noting that $I_2 = -K_2$, one can rewrite $C_1(r)$ as

$$C_1(r) = \frac{1}{1+\beta}\left[K_1 + Y_1 + \frac{1-\alpha}{1+r}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}}\right].$$

It is then straightforward to compute saving as $S_1(r) = Y_1 - C_1(r)$, and check that $S_1'(r) > 0$:

$$S_1'(r) = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \frac{1}{1+\beta}\left[\frac{\alpha+r}{r(1+r)^2}\right] > 0.$$

Refer to sections 1.3.2.2 and 1.3.4 in the book for analysis of the slope of the saving schedule in the case of a general utility function. In this particular exercise where $u(C) = \log(C)$, the intertemporal substitution elasticity σ is equal to 1. Thus the substitution and income effects cancel each other and only the wealth effect (which raises saving) remains. The slope of the saving schedule therefore is unambiguously signed.

4. (a) Recalling eq. (25) of Chapter 1 in the book,

$$C_2 = (1+r)^{\sigma} \beta^{\sigma} C_1,$$

one sees that $C_1 = C_2$ as σ goes to 0.

(b) The expression for $C_1(r)$ can be obtained using the intertemporal budget constraint together with $C_1 = C_2$. In a two-period model

$$CA_2 = Y_2 + r(Y_1 - C_1) - C_2 = -(Y_1 - C_1) = -CA_1,$$

one can verify that $C_2(r) = C_1(r)$ by substituting the expression for $C_1(r)$ into the second of the preceding equalities and solving for $C_2(r)$.

(c) Differentiating the expression for $C_1(r)$ obtained in part b, one has

$$\frac{\mathrm{d}C_1}{\mathrm{d}r} = \frac{Y_1 - Y_2}{(2+r)^2}.$$

When $\sigma = 0$ the pure substitution effect of an increase in r is equal to zero, so that the only effect is the intertemporal terms of trade effect. Given that

$$S_1 = Y_1 - C_1 = \frac{Y_1 - Y_2}{2+r},$$

the country will run a current account surplus if and only if $Y_1 > Y_2$. In that case an increase in r represents a terms of trade improvement, and consequently C_1 increases.

(d) Under the postulated utility function the intertemporal Euler condition is

$$C_1^{-1/\sigma} = (1+r)\beta^{1/\sigma} C_2^{-1/\sigma},$$

or

$$C_2 = (1+r)^{\sigma}\beta C_1.$$

As $\sigma \to 0$, the Euler equation approaches $C_2 = \beta C_1$.

5. The global equilibrium condition is

$$Y_1 + Y_1^* = C_1 + C_1^*.$$

Assume that Y_1 changes. In the notation of section 1.3.4, with $R = 1/(1+r)$,total differentiation of the global equilibrium condition gives

$$1 = \frac{\partial C_1(R, W_1)}{\partial W_1}\frac{\mathrm{d}W_1}{\mathrm{d}Y_1} + \left[\frac{\mathrm{d}C_1(R, W_1)}{\mathrm{d}R} + \frac{\mathrm{d}C_1^*(R, W_1^*)}{\mathrm{d}R}\right]\frac{\mathrm{d}R}{\mathrm{d}Y_1},$$

from which

$$\frac{\mathrm{d}R}{\mathrm{d}Y_1} = \frac{1 - \dfrac{\partial C_1(R, W_1)}{\partial W_1}\dfrac{\mathrm{d}W_1}{\mathrm{d}Y_1}}{\left[\dfrac{\mathrm{d}C_1(R, W_1)}{\mathrm{d}R} + \dfrac{\mathrm{d}C_1^*(R, W_1^*)}{\mathrm{d}R}\right]}$$

follows. Since $\mathrm{d}W_1/\mathrm{d}Y_1 = 1$ and $\partial C_1(R, W_1)/\partial W_1 < 1$ if both periods' consumptions are normal goods, a rise in Y_1 lowers the world interest rate r (i.e., $\mathrm{d}R/\mathrm{d}Y_1 > 0$) provided a fall in r (rise in R) raises total world consumption. As long as the condition for stability (in the Walrasian sense) of the world market equilibrium is satisfied, however, the last condition holds. The effect of a rise in Y_1^* is symmetric, whereas that of a rise in Y_2 follows from

$$\frac{\mathrm{d}R}{\mathrm{d}Y_2} = \frac{-\left(\dfrac{1}{1+r}\right)\dfrac{\partial C_1(R, W_1)}{\partial W_1}}{\left[\dfrac{\mathrm{d}C_1(R, W_1)}{\mathrm{d}R} + \dfrac{\mathrm{d}C_1^*(R, W_1^*)}{\mathrm{d}R}\right]}.$$

6. (a) Date 1 equilibrium (with internationally identical isoelastic preferences) is

$$0 \;=\; Y_1 - \frac{1}{1+(1+r)^{\sigma-1}\beta^{\sigma}}\left[Y_1 - I_1 + \frac{A_2F(I_1) + I_1}{1+r}\right] - I_1 \tag{4}$$

$$+ Y_1^* - \frac{1}{1+(1+r)^{\sigma-1}\beta^{\sigma}}\left[Y_1^* - I_1^* + \frac{A_2^* F(I_1)^* + I_1^*}{1+r}\right] - I_1^*,$$

where $r = A_2 F'(I_1)$ determines investment in Home (with a similar equation in Foreign) and where we have substituted $I_2 = -I_1$, and similarly for Foreign. (We assume $K_1 = 0$ and depreciation $= 0$.). We first differentiate to find the equilibrium change in the interest rate, dr. Using the equation for dC_1/dr on p. 29 of Chapter 1 and the envelope theorem, we see that the differential of the equilibrium condition above is

$$\begin{aligned} 0 \quad = \quad & -\frac{CA_1 - \sigma C_2/(1+r)}{1+r+(C_2/C_1)}\mathrm{d}r + \frac{CA_1 + \sigma C_2^*/(1+r)}{1+r+(C_2^*/C_1^*)}\mathrm{d}r \\ & -\frac{F(I_1)/(1+r)}{1+(1+r)^{\sigma-1}\beta^{\sigma}}\mathrm{d}A_2 \\ & -\left(\frac{\partial I_1}{\partial r} + \frac{\partial I_1^*}{\partial r}\right)\mathrm{d}r - \frac{\partial I_1}{\partial A_2}\mathrm{d}A_2. \end{aligned}$$

Solving for dr (and using the Euler equation to eliminate C_2/C_1) gives

$$\mathrm{d}r = \frac{F(I_1) + [1+r+(1+r)^{\sigma}\beta^{\sigma}]\,\dfrac{\partial I_1}{\partial A_2}}{\dfrac{\sigma}{1+r}(C_2 + C_2^*) - [1+r+(1+r)^{\sigma}\beta^{\sigma}]\left(\dfrac{\partial I_1}{\partial r} + \dfrac{\partial I_1^*}{\partial r}\right)}\mathrm{d}A_2 > 0. \quad (5)$$

The effect on Foreign's current account $CA_1^* = Y_1^* - C_1^* - I_1^*$ is

$$\mathrm{d}CA_1^* = -\frac{CA_1^* - \sigma C_2^*/(1+r)}{1+r+(1+r)^{\sigma}\beta^{\sigma}}\mathrm{d}r - \frac{\partial I_1^*}{\partial r}\mathrm{d}r.$$

Notice that $\partial I_1^*/\partial r < 0$. Accordingly, if $CA_1^* < 0$ (implying that *Home* has a date 1 surplus, $CA_1 > 0$), Foreign's date 1 current account improves whereas Home's worsens. For Foreign, the terms of trade and substitution effects of the rise in r both work to raise saving in this case, and since investment I_1^*must fall, CA_1^* rises and CA_1 falls.

(b) To see what happens to when A_2^* rises, note that the positions of Home and Foreign are simply reversed, but since $CA_1 > 0$, the rise in the interest rate has a positive terms of trade effect on Home that tends to increase C_1 and reduce S_1. So we need (at the least) to calculate

$$\mathrm{d}CA_1 = -\frac{CA_1 - \sigma C_2/(1+r)}{1+r+(1+r)^\sigma \beta^\sigma}\mathrm{d}r - \frac{\partial I_1}{\partial r}\mathrm{d}r \tag{6}$$

where $\mathrm{d}r$ is as in eq. (5) above except for the obvious changes:

$$\mathrm{d}r = \frac{F^*(I_1^*) + [1+r+(1+r)^\sigma \beta^\sigma]\,\dfrac{\partial I_1^*}{\partial A_2^*}}{\dfrac{\sigma}{1+r}(C_2 + C_2^*) - [1+r+(1+r)^\sigma \beta^\sigma]\left(\dfrac{\partial I_1}{\partial r} + \dfrac{\partial I_1^*}{\partial r}\right)}\mathrm{d}A_2^*.$$

The necessary and sufficient condition for CA_1 to rise is clearly, from eq. (6) above,

$$\frac{\sigma C_2}{1+r} - [1+r+(1+r)^\sigma \beta^\sigma]\,\frac{\partial I_1}{\partial r} > CA_1.$$

But there is no reason this has to hold (we can guarantee it fails by making σ and $\partial I_1/\partial r$ very small while making Y_1 as large as it has to be to generate a positive CA_1 of sufficient magnitude). Summary: If Home has a current account surplus on date 1 when A_2 rises, its date 1 surplus falls (and Foreign's date 1 deficit shrinks, or course). If A_2^* rises in the same circumstances, Home's date 1 surplus may rise or fall and Foreign's date 1 deficit (correspondingly) may rise or fall.

7. (a) The intertemporal Euler equation is $\exp(-C_1/\gamma) = (\beta/R)\exp(-C_2/\gamma)$. Taking logs and solving yields

$$C_2 = C_1 + \gamma \log(\beta/R).$$

(b) By the intertemporal budget constraint, $C_2 = (1/R)(Y_1 - C_1) + Y_2$. Substituting this into the preceding Euler equation and solving yields

$$C_1 = \frac{Y_1 + RY_2 - \gamma R \log(\beta/R)}{1+R} = \frac{W_1 - \gamma R \log(\beta/R)}{1+R}.$$

(c) Total differentiation with respect to R yields

$$\begin{aligned}\frac{\mathrm{d}C_1}{\mathrm{d}R} &= \frac{Y_2 - \gamma\log(\beta/R) + \gamma}{1+R} - \frac{W_1 - \gamma R\log(\beta/R)}{(1+R)^2} \\ &= -\frac{C_1}{1+R} + \frac{Y_2}{1+R} + \frac{\gamma}{1+R}\left[1 - \log(\beta/R)\right].\end{aligned}$$

(d) We have

$$\sigma(C) = \frac{-\exp(-C/\gamma)}{-(C/\gamma)\exp(-C/\gamma)} = \frac{\gamma}{C}.$$

(e) Using the Euler equation in part a, one can substitute for C_1 above to get

$$\frac{\mathrm{d}C_1}{\mathrm{d}R} = \frac{Y_2 - C_2}{1+R} + \frac{\sigma(C_2)C_2}{1+R}.$$

What do these terms capture? The first is the terms-of-trade effect, i.e., the wealth effect on $C_1, Y_2/(1+R)$, plus the Slutsky income-effect term $-(\partial C_1/\partial W_1)C_2 = C_2/(1+R)$. [See eq. (30) in Chapter 1.] To see that the final term is the compensated price response of demand (i.e., the pure substitution effect), use the Euler equation to write *lifetime* utility as

$$\begin{aligned}U_1 &= -\gamma\exp(-C_1/\gamma) - \beta\gamma\exp\left[-C_1/\gamma - \log(\beta/R)\right] \\ &= -\gamma\,(1+R)\exp(-C_1/\gamma)\end{aligned}$$

and solve for C_1 as a function of R and U_1:

$$C_1^{\mathrm{H}}(R, U_1) = \gamma\log\left[\gamma(1+R)\right] - \gamma\log(-U_1).$$

From this equation,

$$\frac{\partial C_1^{\mathrm{H}}}{\partial R} = \frac{\gamma}{1+R} = \frac{\sigma(C_2)C_2}{1+R}.$$

8. (a) The Home government maximizes $u(C_1, C_2)$ subject to the national constraint $C_2 = Y_2 + (1+r)(Y_1 - C_1)$ where r is the world interest rate.

Since private Foreigners carry out a similar maximization of $u[C_1^*, Y_2^* + (1 + r)(Y_1^* - C_1^*)]$, their date 1 consumption obeys the Euler equation

$$\begin{aligned} & \partial u\left[C_1^*, Y_2^* + (1+r)(Y_1^* - C_1^*)\right]/\partial C_1^* \\ = \; & (1+r)\partial u\left[C_1^*, Y_2^* + (1+r)(Y_1^* - C_1^*)\right]/\partial C_2^*, \end{aligned}$$

which implicitly yields the function $C_1^*(r)$. Thus, we may regard the Home government as maximizing (with respect to C_1)

$$u\left[C_1, Y_2 + (1+r)(Y_1 - C_1)\right]$$

subject to the goods-market equilibrium condition

$$C_1 + C_1^*(r) = Y_1 + Y_1^*,$$

which implies that C_1 will influence r. Indeed, by substitution into the Home welfare criterion of this last constraint, we may view the Home government as directly choosing the world interest rate r to maximize

$$u\left\{Y_1 + Y_1^* - C_1^*(r), Y_2 + (1+r)\left[C_1^*(r) - Y_1^*\right]\right\}.$$

The first-order condition for this last (unconstrained) maximization with respect to r is:

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}r} &= \frac{\partial u(C_1, C_2)}{\partial C_1} C_1^{*\prime}(r) + \frac{\partial u(C_1, C_2)}{\partial C_2}(1+r)C_1^{*\prime}(r) + \frac{\partial u(C_1, C_2)}{\partial C_2}\left[C_1^*(r) - Y_1^*\right] \\ &= 0. \end{aligned}$$

The trick now is to recognize that if the government is manipulating the world interest rate through a borrowing tax τ, the Euler equation for Home's *private* agents will be

$$\frac{\partial u(C_1, C_2)}{\partial C_1} = (1+r)(1+\tau)\frac{\partial u(C_1, C_2)}{\partial C_2},$$

so that we may write the Home *government's* Euler equation (the displayed equation that precedes the Home private Euler equation) as

$$\frac{\mathrm{d}u}{[\partial u(C_1, C_2)/\partial C_2]} = -\tau(1+r)C_1^{*\prime}(r)\mathrm{d}r + [C_1^*(r) - Y_1^*]\,\mathrm{d}r = 0 \qquad (7)$$

after division by $\partial u(C_1, C_2)/\partial C_2$. Remembering that the country wishes to lower its first-period borrowing cost $(1+r)\,[Y_1^* - C_1^*(r)] > 0$ by reducing the world real interest rate ($\mathrm{d}r < 0$), we see that the second summand above, $[C_1^*(r) - Y_1^*]\,\mathrm{d}r > 0$, is the direct gain in real income from a lower borrowing cost. The first summand is a welfare loss due to the fact that first-period borrowing from Foreigners is being reduced [Foreign consumption is rising, $C_1^{*\prime}(r)\mathrm{d}r > 0$], despite the fact that the tax wedge τ makes the domestic marginal utility of first-period consumption higher than its true, world opportunity cost. As usual, the optimal tax exactly balances the marginal domestic terms-of-trade gain against the marginal domestic welfare loss. The rest is merely a matter of manipulating definitions. Recall that $IM_2^* \equiv C_2^* - Y_2^*$ (there is no investment in this example), so that the Foreign budget constraint yields

$$IM_2^*(r) = (1+r)\,[Y_1^* - C_1^*(r)]\,.$$

Therefore,

$$\begin{aligned}\zeta^* &\equiv (1+r)IM_2^{*\prime}(r)/IM_2^*(r) = \frac{-(1+r)C_1^{*\prime}(r) + [Y_1^* - C_1^*(r)]}{Y_1^* - C_1^*(r)} \\ &= 1 + \frac{1}{\tau},\end{aligned}$$

where τ is the optimal tax derived from solving the Home government first-order condition, eq. (7). This last equation, however, implies that

$$\tau = \frac{1}{\zeta^* - 1},$$

as part a of this problem asserts.

(b) The condition $\zeta^* > 1$ can be understood as follows. If Foreign demand for second-period imports of consumption is inelastic ($\zeta^* \leq 1$), then a rise in their price (a fall in the world interest rate $1+r$) elicits a proportionally smaller fall in demand. But since $IM_2^*(r) = (1+r)\left[Y_1^* - C_1^*(r)\right]$, this means that $\left[Y_1^* - C_1^*(r)\right]$ alone actually must *rise* (stay constant for $\zeta^* = 1$) when $1+r$ falls. According to the reasoning following eq. (7), however, Home welfare thus unambiguously rises when $\zeta^* \leq 1$ and the borrowing tax is raised. This means that a maximizing Home government will never find itself at a position where $\zeta^* \leq 1$. (Think of this condition as analogous to the familiar result that a monopolist always operates on the elastic stretch of the demand curve it faces.)