Foundations of International Macroeconomics[1]

Workbook[2]

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Chapter 10 Solutions

1. (a) With a positive steady-state gross money supply growth rate of $1+\mu$, eq. (26) in Chapter 10 is replaced by

$$\overline{m}_0 = \overline{m}_0^* = \frac{\chi}{1 - \frac{1}{(1+\mu)(1+\delta)}}\overline{y}_0, \tag{26$'$}$$

where (for this problem only) $m \equiv M/P$ denotes the real money supply. The value for $\overline{y}_0$ is the same as in the case with zero steady-state money growth. The log-linear version of the money demand equation becomes

$$\mathsf{m}_{\text{real},t} = \mathsf{c}_t - \frac{\delta}{(1+\mu)(1+\delta)^2 - (1+\delta)}\mathsf{r}_{t+1} - \frac{1}{(1+\mu)(1+\delta) - 1}\Delta\mathsf{p}_{t+1}, \tag{37$'$}$$

where $\mathsf{m}_{\text{real},t} = \mathrm{d}m_t/\overline{m}$ denotes the percentage deviation of real money balances from their steady-state level, $\Delta\mathsf{p}_t \equiv [(P_t/P_{t-1})/(1+\mu)] - 1$ denotes the percentage deviation of inflation from its (gross) steady-state level of $1+\mu$, and the other variables are as defined in the text. The foreign counterpart to (37$'$) is

$$\mathsf{m}^*_{\text{real},t} = \mathsf{c}^*_t - \frac{\delta}{(1+\mu)(1+\delta)^2 - (1+\delta)}\mathsf{r}_{t+1} - \frac{1}{(1+\mu)(1+\delta) - 1}\Delta\mathsf{p}^*_{t+1}. \tag{38$'$}$$

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Denote by $\boldsymbol{\mu}_t$ (a boldface μ) the percentage deviation of nominal money growth from its steady-state value. Note that

$$\boldsymbol{\mu}_t = \Delta \mathsf{m}_{\mathrm{real},t} + \Delta \mathsf{p}_t.$$

Also, by consumption-based purchasing power parity,

$$\Delta \mathsf{p}_t = \Delta \mathsf{p}_t^* + \Delta \mathsf{e}_t,$$

where $\Delta \mathsf{e}_t$ is the percentage deviation of the *growth rate* of the nominal exchange rate, $\mathcal{E}_t/\mathcal{E}_{t-1}$, from its initial date-zero steady-state value of unity (recall that $\mu = \mu^*$). It is then possible to derive

$$\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^* - \Delta \mathsf{e}_t = \mathsf{c}_t - \mathsf{c}_t^* - \left(\mathsf{c}_{t-1} - \mathsf{c}_{t-1}^*\right) - \frac{1}{(1+\mu)(1+\delta) - 1} \left(\Delta \mathsf{e}_{t+1} - \Delta \mathsf{e}_t\right). \tag{39$'$}$$

Solutions for steady-state equilibrium inflation and nominal exchange rate growth follow immediately from eqs. (37′)-(39′):

$$\overline{\boldsymbol{\mu}} - \overline{\Delta \mathsf{p}} = 0, \tag{50$'$}$$

$$\overline{\boldsymbol{\mu}}^* - \overline{\Delta \mathsf{p}}^* = 0, \tag{51$'$}$$

$$\overline{\boldsymbol{\mu}} - \overline{\boldsymbol{\mu}}^* = \overline{\Delta \mathsf{e}}. \tag{52$'$}$$

(b) Assume a permanent unanticipated rise in the home rate of money growth occurring on date 1, with prices preset a period in advance and adjusting to their flexible-price level after one period, absent new shocks. Given that in the initial steady state the exchange rate is expected to remain constant (because initially, $\mu = \mu^*$), it follows that

$$\Delta \mathsf{e} = \mathsf{e},$$

where e is the percentage deviation of the nominal exchange rate from its preshock steady state level—i.e., its level along the economy's steady-state path. (As in the text, sans serif variables with overbars denote new post-shock steady-state values, for period 2 and beyond. Variables without bars denote postshock date 1 values; thus $\Delta \mathsf{e} \equiv \mathsf{e}_1 - \mathsf{e}_0$.) Note that $\overline{\Delta \mathsf{e}}$ can also be interpreted as the percentage by which the nominal exchange rate would change on impact if prices were fully flexible, so that

$$\overline{\Delta \mathsf{e}} = \mathsf{e}^{flex},$$

(That is, e^{flex} is the percentage deviation of $\mathcal{E}^{flex}$ from its pre-shock steady state level, with $\mathcal{E}^{flex}$ being the nominal exchange that would obtain on impact if output prices were fully flexible.) It is then possible to rewrite eq. (39′) as:

$$\mathsf{e} = \mathsf{e}^{flex} - \frac{(1+\mu)(1+\delta) - 1}{(1+\mu)(1+\delta)} (\mathsf{c} - \mathsf{c}^*), \tag{60′}$$

where $\mathsf{e}^{flex} = \overline{\boldsymbol{\mu}} - \overline{\boldsymbol{\mu}}^*$, with $\overline{\boldsymbol{\mu}}^* = 0$ and $\mathsf{c} - \mathsf{c}^* = \overline{\mathsf{c}} - \overline{\mathsf{c}}^*$. Figure 10.1 shows eq. (60′) as the downward sloping **MM** schedule. Notice that the short-run nominal exchange rate, e, is going to be less than the value that would obtain if prices were fully flexible, e^{flex}, because the rise in the Home rate of money growth relative to Foreign's entails a short-run increase in the consumption growth differential.

As in the text, it is possible to derive a second schedule in e and $\mathsf{c} - \mathsf{c}^*$ using the short-run equilibrium conditions other than the money demand equations, together with eq. (45) of Chapter 10, and recalling that on impact $\Delta \mathsf{e} = \mathsf{e}$. Denote by $\mathcal{P}_t$ the percentage deviation of $p_t(h)/P_t$ from its pre-shock steady-state level of unity, and by $\Delta \mathsf{p}_t(h)$ the percentage deviation of $p_t(h)/p_{t-1}(h)$ from its pre-shock steady-state level of $1+\mu$. It is easy to show that $\Delta \mathsf{p} = (1-n)\,\mathsf{e}$, and $\mathcal{P} - \mathcal{P}^* = -\mathsf{e}$. Following the same steps as in the chapter, one then obtains

$$\mathsf{e} = \frac{\delta(1+\theta) - 2\theta}{\delta(\theta^2 - 1)} (\mathsf{c} - \mathsf{c}^*), \tag{64′}$$

which is the upward-sloping schedule **GG** in figure 10.1. The **GG** locus has a positive slope because Home's consumption growth can rise relative to Foreign's in the short-run only if the growth in the nominal exchange rate increases, allowing Home's output to rise relative to Foreign's. The intersection of the two schedules gives the equilibrium nominal exchange rate at the time of the shock. Note that the level of $\mathsf{c} - \mathsf{c}^*$ given by the diagram is permanent, but eq. (52′) must be used to calculate nominal exchange rate growth after the initial, sticky-price period.

2. The nominal home-currency price of a nontraded good is P_{N}, and that of a traded good is P_{T}. Therefore, the real price of nontraded in terms of traded goods is $\rho \equiv P_{\text{N}}/P_{\text{T}}$, and the consumption-based price index corresponding to the utility function specified in the problem is $\rho^{1-\gamma}/\gamma^{\gamma}(1-\gamma)^{1-\gamma}$ (expressed in units of traded goods). Since P_{N} is fixed in the short run, and since an unanticipated money-supply increase depreciates the domestic currency making P_{T} rise, it simultaneously lowers $\rho^{1-\gamma}$. The resulting change in the log of the consumption-based price index (measured in tradables) is approximately $-(1-\gamma)\hat{P}_{\text{T}}$.

Notice that this change corresponds (albeit with the *opposite* sign) to the change in the country's real exchange rate, $q = \mathcal{E}P^*/P$, where P is the home consumer price index and P^* is the rest-of-world consumer price index measured in foreign currency units. Since it is assumed that purchasing power parity holds for traded goods ($P_{\text{T}} = \mathcal{E}P_{\text{T}}^*$),

$$q = \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}P^*/\mathcal{E}P_{\text{T}}^*}{P/P_{\text{T}}} = \frac{P^*/P_{\text{T}}^*}{P/P_{\text{T}}}.$$

Because

$$P/P_{\text{T}} = \frac{\rho^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

(given the assumed utility function) and because P^*/P_{T}^* is not affected by the shock in the small country, the absolute change in $\rho^{1-\gamma}$ equals the increase in q—which is a real depreciation of the domestic currency.

The consumption-based real interest rate is given by

$$1 + r^{\mathrm{C}}_{t+1} = \frac{(1+r)\left(P_{\mathrm{N},t}/P_{\mathrm{T},t}\right)^{1-\gamma}}{\left(P_{\mathrm{N},t+1}/P_{\mathrm{T},t+1}\right)^{1-\gamma}}.$$

(Refer to section 4.4.1.3 in the book to see how this expression is derived. The world interest rate r is the own-rate on tradables in section 10.2.) Money is completely neutral in the long run in this particular model (but only because the special structure of the model implies that an unanticipated money shock has no current-account effects.) Therefore, $P_{\mathrm{N},t+1}/P_{\mathrm{T},t+1}$ is unaffected by the money shock and the direction in which the consumption-based real interest rate moves in the short run is the same as the direction in which $P_{\mathrm{N},t}/P_{\mathrm{T},t}$ moves. So, in fact, an unanticipated domestic money-supply increase causes a fall in the home real interest rate and a real depreciation of the home currency, just as in the Dornbusch model. It is easy to see that this result extends to money growth shocks, since it is still the case that any *real* effect on $P_{\mathrm{N},t}/P_{\mathrm{T},t}$ dies out after just one period.

3. Let p_y be the home-currency price of the single good exported to the rest of the world, P the home-currency price of the imported good. Assume that $P = \mathcal{E}P^*$, where P^* is the rest-of-world price index and $\mathcal{E}$ is the home-currency price of the rest-of-world currency. One can then rewrite the demand curve faced by the small country as:

$$y^d = \left(\frac{p_y}{P}\right)^{-\theta} C^{\mathrm{W}}.$$

The utility function of the small country's representative agent is

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s + \chi \log \frac{M_s}{P_s} - \frac{\kappa}{2} y_s^2\right], \tag{1'}$$

where C is consumption of the single imported good. The period budget constraint is

$$P_t B_{t+1} + M_t = (1+r) P_t B_t + M_{t-1} + p_{y,t} y_t - P_t C_t - P_t \tau_t, \tag{8'}$$

where r is the constant world net interest rate, with $(1+r)\beta = 1$. Throughout, a change in the foreign-currency price of the good produced by the small country is assumed to have, *ceteris paribus*, a negligible effect on the foreign-currency world price index P^*. The first-order conditions for the maximization problem of the small-country representative agent are

$$C_{t+1} = C_t, \tag{13'}$$

$$\frac{M_t}{P_t} = \chi C_t \left(\frac{1 + i_{t+1}}{i_{t+1}} \right), \tag{14'}$$

$$y_t^{\frac{\theta+1}{\theta}} = \frac{\theta - 1}{\theta \kappa} \left(C_t^{\mathrm{W}} \right)^{\frac{1}{\theta}} \frac{1}{C_t}, \tag{15'}$$

where $1 + i_{t+1} = (1+r) P_{t+1}/P_t$ and the usual transversality condition must hold. Assuming an initial symmetric steady-state where $\overline{p}_y = \overline{\mathcal{E}}\, \overline{P}^*$ and $\overline{C} = \overline{C}^{\mathrm{W}}$, so that $\overline{p}_y = \overline{P}$ and $\overline{B} = 0$, and log-linearizing around that steady-state, one obtains the following log-linear versions of the first-order conditions,

$$\mathsf{c}_{t+1} = \mathsf{c}_t, \tag{35'}$$

$$\mathsf{m}_t - \mathsf{e}_t = \mathsf{c}_t - \frac{1}{r} \left(\mathsf{e}_{t+1} - \mathsf{e}_t \right), \tag{37'}$$

$$(\theta + 1)\, \mathsf{y}_t = -\theta \mathsf{c}_t, \tag{33'}$$

where it is assumed that $\mathsf{c}^{\mathrm{W}} = \mathsf{p}^* = 0$ on every date, so that $\mathsf{p}_t = \mathsf{e}_t$, and $\mathsf{y}_t = \theta \left(\mathsf{e}_t - \mathsf{p}_{y,t} \right)$. The log-linearized version of the economy-wide resource constraint is then

$$\mathsf{b}_{t+1} = (1+r)\, \mathsf{b}_t + (\theta - 1) \left(\mathsf{e}_t - \mathsf{p}_{y,t} \right) - \mathsf{c}_t.$$

Following the same steps as in the text, one can show straightforwardly that in the steady-state,

$$\overline{\mathsf{c}} = \frac{1+\theta}{2\theta} r \overline{\mathsf{b}}, \tag{45'}$$

$$\overline{\mathsf{p}}_{y,t} - \overline{\mathsf{e}}_t = \frac{1}{2\theta} r \overline{\mathsf{b}}, \tag{46'}$$

$$\overline{\mathsf{m}} - \overline{\mathsf{e}} = \overline{\mathsf{c}}. \tag{52$'$}$$

If one assumes that the small-country-currency price p_y of the export good is set one period in advance, and reverts to its flexible-price level after a single period absent new shocks, then, given that $\overline{\mathsf{c}} = \mathsf{c}$ and $\overline{\mathsf{m}} = \mathsf{m}$, it follows from eq. (37$'$) that $\mathsf{e} = \overline{\mathsf{e}}$. Moreover, since in the short run $\mathsf{y} = \theta\mathsf{e}$, and thus $\overline{\mathsf{b}} = (\theta - 1)\,\mathsf{e} - \mathsf{c}$, one derives the following schedule in e and c,

$$\mathsf{e} = \frac{r\,(1+\theta) + 2\theta}{r\,(\theta^2 - 1)}\mathsf{c}, \tag{64$'$}$$

which, together with the schedule $\mathsf{e} = \mathsf{m} - \mathsf{c}$, gives the equilibrium exchange rate

$$\mathsf{e} = \frac{r\,(1+\theta) + 2\theta}{\theta r\,(1+\theta) + 2\theta}\mathsf{m}. \tag{65$'$}$$

Because $\delta = (1-\beta)/\beta = r$, this last equation is the same as eq. (65) in the text when the level of the money supply in the rest of the world is constant.

4. [In the cash-in-advance constraint given in the statement of this exercise, $C^j_{\mathrm{N},t}(z)$ should be $c^j_{\mathrm{N},t}(z)$.] Individual j's period budget constraint is:

$$\begin{aligned} P_{\mathrm{T},t}B^j_{t+1} + M^j_t \;&=\; P_{\mathrm{T},t}(1+r)B^j_t + M^j_{t-1} + p_{\mathrm{N},t}(j)y_{\mathrm{N},t}(j) \\ &\quad + P_{\mathrm{T},t}\bar{y}_{\mathrm{T}} - P_{\mathrm{N},t}C^j_{\mathrm{N},t} - P_{\mathrm{T},t}C^j_{\mathrm{T},t} - P_{\mathrm{T},t}\tau_t. \end{aligned} \tag{1}$$

A nontraded-goods producer j faces the demand curve

$$y^d_{\mathrm{N}}(j) = \left[\frac{p_{\mathrm{N}}(j)}{P_{\mathrm{N}}}\right]^{-\theta} C^{\mathrm{A}}_{\mathrm{N}}. \tag{2}$$

The first-order conditions are found by maximizing the lifetime utility function subject to (1), (2) and the contemporaneous cash-in-advance constraint:

$$B_{t+1}: \qquad C_{\mathrm{T},t} = C_{\mathrm{T},t+1},$$

$$C_{\mathrm{N},t}: \qquad C_{\mathrm{N},t} = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{P_{\mathrm{T},t}}{P_{\mathrm{N},t}}\right)C_{\mathrm{T},t},$$

$$y_{\mathrm{N},t}: \qquad y_{\mathrm{N},t}^{\frac{\theta+1}{\theta}} = \frac{(\theta-1)}{\theta\kappa C_{\mathrm{T},t}}\left(\frac{\gamma P_{\mathrm{N},t}C^{\mathrm{A}\,\frac{1}{\theta}}_{\mathrm{N},t}}{P_{\mathrm{T},t}}\right).$$

Substitute for $C_{\mathrm{T},t}$ in the first-order condition for $y_{\mathrm{N},t}$ using the first-order condition for $C_{\mathrm{N},t}$ to obtain

$$y_{\mathrm{N},t}^{\frac{\theta+1}{\theta}} = \left[\frac{(\theta-1)(1-\gamma)}{\theta\kappa}\right]\frac{C_{\mathrm{N},t}^{\mathrm{A}\,\frac{1}{\theta}}}{C_{\mathrm{N},t}}. \tag{3}$$

Once consumers have determined the amount of traded and nontraded goods they wish to consume, and the price at which they will sell their output, the cash-in-advance constraint determines the amount of money they wish to hold. This is because money does not enter the utility function. As long as the nominal interest rate on bonds is positive, people will never want to hold any money in excess of what they require to finance current consumption.

(a) *Flexible price case*: In the symmetric market equilibrium, $C_{\mathrm{N},t} = y_{\mathrm{N},t} = C_{\mathrm{N},t}^{\mathrm{A}}$ for every nontradable good z. Thus equation (3) implies that in the flexible-price equilibrium,

$$\bar{y}_{\mathrm{N}} = \left[\frac{(\theta-1)(1-\gamma)}{\theta\kappa}\right]^{\frac{1}{2}}. \tag{4}$$

(b) The monopoly level of output of each nontraded good is too low. As discussed in the text on p. 668, a planner equates the marginal utility of composite nontradables consumption with the marginal welfare cost of higher output in terms of forgone leisure. Assuming the planner gives all agents equal weight, his problem can be written as that of maximizing

$$U_t = \sum_{s=t}^{\infty}\beta^{s-t}\left[\gamma\log\bar{y}_{\mathrm{T}} + (1-\gamma)\log y_{\mathrm{N},s} - \frac{\kappa}{2}\left(y_{\mathrm{N},s}\right)^2\right].$$

(The planner internalizes the constraints that in a symmetric allocation, $y_{\mathrm{N},t} = C_{\mathrm{N},t} = C_{\mathrm{N},t}^{\mathrm{A}}$ and $\bar{y}_{\mathrm{T}} = C_{\mathrm{T},t}$.) The first-order condition with respect to $y_{\mathrm{N},s}$ gives the optimal level of nontraded goods output that the planner will choose,

$$\frac{(1-\gamma)}{y_{\mathrm{N}}} = \kappa y_{\mathrm{N}},$$

which yields

$$\bar{y}_{\mathrm{N}}^{\mathrm{PLAN}} = \left[\frac{(1-\gamma)}{\kappa}\right]^{\frac{1}{2}}. \tag{5}$$

The planner will therefore choose the level of output at which the marginal utility from consumption of nontradables is equal to the marginal cost the leisure forgone in producing them. Plainly this output level exceeds that in part a, see eq. (4).

(c) The cash-in-advance constraint holds with equality in equilibrium:

$$M_t = P_{\mathrm{T},t}\,\bar{y}_{\mathrm{T}} + P_{\mathrm{N},t}\,y_{\mathrm{N},t}. \tag{6}$$

Furthermore,

$$C_{\mathrm{T},t} = \bar{y}_{\mathrm{T}}. \tag{7}$$

In the flexible-price case, the money price of nontraded goods is found by combining the first-order condition for $C_{\mathrm{N},t}$ with eqs. (6) and (7):

$$P_{\mathrm{N},t} = \frac{(1-\gamma)M_t}{y_{\mathrm{N},t}}. \tag{8}$$

Given the symmetry of the model, the period t money price of every non-traded good will be set at the level $\bar{p}_{\mathrm{N},t}$ at which, in the absence of monetary surprises, each producer's output would be given by $\bar{y}_{\mathrm{N}}$ in eq. (4). Thus,

$$\bar{p}_{\mathrm{N},t} = \bar{P}_{\mathrm{N},t} = \frac{(1-\gamma)M_t^{\mathrm{e}}}{\bar{y}_{\mathrm{N}}}. \tag{9}$$

Given temporarily fixed nontraded goods prices, short-run output is demand determined, that is, $y_{\mathrm{N},t} = C_{\mathrm{N},t}$. Using this result, together with (8) and (9), we obtain the solution for $y_{\mathrm{N},t}$:

$$y_{\mathrm{N},t} = \frac{M_t\,\bar{y}_{\mathrm{N}}}{M_t^{\mathrm{e}}} \tag{10}$$

(d) Period t monetary policy affects only period t welfare in the one-shot game. Also, $C_{\mathrm{T},t} = \bar{y}_{\mathrm{T}}$ in equilibrium. The monetary authorities therefore set M_t to maximize the expression specified in the problem since the other elements of the representative agent's objective function are exogenous.

(e) Eliminate $C_{\text{N},t}$ from the monetary authorities' objective function using eq. (10) above. Observe also that the price of tradables always moves proportionally to the money supply, *ceteris paribus*, because the first-order condition for $C_{\text{N},t}$ and eq. (6) together imply that

$$P_{\text{T},t} = \frac{\gamma M_t}{\bar{y}_{\text{T}}}.$$

One therefore can express the authorities' maximization problem as:

$$\max_{M_t} \left\{ (1-\gamma)\left(\log M_t + \log \bar{y}_{\text{N}} - \log M_t^{\text{e}}\right) - \frac{\kappa}{2}\left(\frac{M_t}{M_t^{\text{e}}}\bar{y}_{\text{N}}\right)^2 - \frac{\chi}{2}\left(\frac{M_t}{M_{t-1}}\right)^2 \right\}.$$

Take the derivative with respect to M_t to get the solution for the one-shot-game equilibrium level of money growth:

$$\frac{(1-\gamma)}{M_t} - \kappa\left(\frac{\bar{y}_{\text{N}}}{M_t^{\text{e}}}\right)^2 M_t - \left(\frac{\chi}{M_{t-1}^2}\right) M_t = 0.$$

Now impose the usual condition for a "time consistent" equilibrium, $M_t^{\text{e}} = M_t$, which implies that $y_{\text{N},t} = \bar{y}_{\text{N}}$. Making use of eq. (4) for steady-state output $\bar{y}_{\text{N}}$, we obtain the solution for equilibrium inflation

$$\frac{M_t}{M_{t-1}} = \frac{P_{\text{T},t}}{P_{\text{T},t-1}} = \frac{P_{\text{N},t}}{P_{\text{N},t-1}} = \left(\frac{1-\gamma}{\chi\theta}\right)^{\frac{1}{2}}. \tag{11}$$

But the right-hand side of eq. (11) above can be written in the alternative form $\left\{\kappa\left[(\bar{y}_{\text{N}}^{\text{PLAN}})^2 - (\bar{y}_{\text{N}})^2\right]/\chi\right\}^{\frac{1}{2}}$; to see why, simply substitute for $\bar{y}_{\text{N}}$ and $\bar{y}_{\text{N}}^{\text{PLAN}}$ using eqs. (4) and (5) from parts a and b.